Proyecciones (Antofagasta. On line) | vol. 39, n. 5 (2020) | pp. 1191-1200.

10.22199/issn.0717-6279-2020-05-0073

PROYECCIONES
Journal of Mathematics

REVISTAS CIENTÍFICAS de la Universidad Católica del Norte. revistas.ucn.cl

ISSN 0717-6279 (On line)

Bioperation approach to Przemski's decomposition theorems

C. Carpintero[1] orcid.org/0000-0003-3831-952X
R. Nirmala[2] orcid.org/0000-0003-0114-6370
N. Rajesh[3]
E. Rosas[4] orcid.org/0000-0001-8123-9344

[1]Corporación Universitaria del Caribe-CECAR, Sincelejo, Colombia.
carpintero.carlos@gmail.com
Rajah Serfoji Government College (Autonomous). Dept. of Mathematics, Namakkal, TN, India.
[2] nirmala.karthik143@gmail.com; [3] nrajesh@gmail.com
[4] Universidad de la Costa, Dept. de Ciencias Naturales y Exactas, Barranquilla, Colombia.
ennisrafael@gmail.com

Received: July 2019 | Accepted: November 2019

Abstract:

Przemski introduced $D(\alpha, s)$-set, $D(\alpha, b)$-set, $D(p, sp)$-set, $D(p, b)$-set and $D(b, sp)$-set to obtain several decompositions of continuity. In this paper, we extend these sets via bioperation and obtain new decompositions of continuity.

Keywords: Topological spaces; $\gamma \vee \gamma'$-open set

MSC (2020): 54A05, 54C08.

Cite this article as (IEEE citation style):

C. Carpintero, R. Nirmala, N. Rajesh, and E. Rosas, "Bioperation approach to Przemski's decomposition theorems", *Proyecciones (Antofagasta, On line)*, vol. 39, no. 5, pp. 1191-1200, Oct. 2020, doi: 10.22199/issn.0717-6279-2020-05-0073.

1. Introduction

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the various modified forms of continuity, separation axioms etc. Utilizing generalized open sets. Kasahara [1] defined the concept of an operation on topological spaces. Ogata and Maki [4] introduced and studied the notion of $\tau_{\gamma\vee\gamma'}$ which is the collection of all $\gamma\vee\gamma'$-open sets in a topological space (X,τ). Przemski in [5], analyze some forms of decomposition of continuous and α-continuous using $D(\alpha,s)$-set, $D(\alpha,b)$-set, $D(p,sp)$-set, $D(p,b)$-set and $D(b,sp)$-set In this paper, we introduce some new types of sets via bioperation and obtain some theorems related with decomposition of continuity.

2. Preiliminaries

The closure and the interior of a subset A of (X,τ) are denoted by (A) and (A), respectively.

Definition 2.1. *[1] Let (X,τ) be a topological space. An operation γ on the topology τ is function from τ on to power set $P(X)$ of X such that $V\subset V^{\gamma}$ for each $V\in\tau$, where V^{γ} denotes the value of τ at V. It is denoted by $\gamma:\tau\to P(X)$.*

Definition 2.2. *A subset A of a topological space (X,τ) is said to be $\gamma\vee\gamma'$-open set [4] if for each $x\in A$ there exists an open neighbourhood U of x such that $U^{\gamma}\cup U^{\gamma'}\subset A$. The complement of $\gamma\vee\gamma'$-open set is called $\gamma\vee\gamma'$-closed. $\tau_{\gamma\vee\gamma'}$ denotes set of all $\gamma\vee\gamma'$-open sets in (X,τ).*

Definition 2.3. *[4] For a subset A of (X,τ), $\tau_{\gamma\vee\gamma'}$-(A) denotes the intersection of all $\gamma\vee\gamma'$-closed sets containing A, that is, $\tau_{\gamma\vee\gamma'}$-$(A)=\cap\{F: A\subset F, X\backslash F\in\tau_{\gamma\vee\gamma'}\}$.*

Definition 2.4. *Let A be any subset of X. The $\tau_{\gamma\vee\gamma'}$-(A) is defined as $\tau_{\gamma\vee\gamma'}$-$(A)=\cup\{U: U$ is a $\gamma\vee\gamma'$-open set and $U\subset A\}$.*

Definition 2.5. *Let (X,τ) be a topological space and A be a subset of X and γ and γ' be operations on τ. Then A is said to be*

1. *$\gamma\vee\gamma'$-α-open if $A\subset\tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)))$*

2. $\gamma \vee \gamma'$*-preopen if* $A \subset \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A))$

3. $\gamma \vee \gamma'$*-semiopen [3] if* $A \subset \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A))$

4. $\gamma\vee\gamma'$*-semipreopen (or* $\gamma\vee\gamma'$*-sp-open) if* $A \subset \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)))$

5. $\gamma \vee \gamma'$*-b-open if* $A \subset \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)) \cup \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A))$

6. $\gamma \vee \gamma'$*-regular open [2] if* $A =_{\gamma\vee\gamma'} (\tau_{\gamma\vee\gamma'}\text{-}(A))$.

$\gamma\vee\gamma'$-semipreinterior of A and denoted by $sp\tau_{\gamma\vee\gamma'}\text{-}(A)$. The complement of a $\gamma \vee \gamma'$-semipreopen set is called a $\gamma \vee \gamma'$-semipreclosed set. It is clear that $sp\tau_{\gamma\vee\gamma'}\text{-}(A) = A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)))$.

Definition 2.6. *Let* (X, τ) *and* (Y, σ) *be two topological spaces and let* $\gamma, \gamma' : \tau \longrightarrow \wp(X)$ *be operations on* τ*. A mapping* $f : (X, \tau) \longrightarrow (Y, \sigma)$ *is said to be* $\gamma\vee\gamma'$*-continuous (resp.* $\gamma\vee\gamma'$*-*α*-continuous,* $\gamma\vee\gamma'$*-precontinuous,* $\gamma \vee \gamma'$*-semicontinuous,* $\gamma \vee \gamma'$*-semiprecontinuous,* $\gamma \vee \gamma'$*-b-continuous) if for each* $x \in X$ *and each open set* V *of* Y *containing* $f(x)$ *there exists a* $\gamma \vee \gamma'$*-open (resp.* $\gamma \vee \gamma'$*-*α*-open,* $\gamma \vee \gamma'$*-preopen,* $\gamma \vee \gamma'$*-semiopen,* $\gamma \vee \gamma'$*-semipreopen,* $\gamma \vee \gamma'$*-b-open) set* U *containing* x *such that* $f(U) \subset V$*.*

3. Some subsets in topological spaces

Definition 3.1. *For a topological space* (X, τ) *with the operations* γ, γ', *we define the following:*

1. $D(\alpha, s) = \{A \subset X : A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(_{\gamma\vee\gamma'}(A))) = A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A))\}$.

2. $D(\alpha, sp) = \{A \subset X : A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(_{\gamma\vee\gamma'}(A))) = A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)))\}$.

3. $D(\alpha, b) = \{A \subset X : A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(_{\gamma\vee\gamma'}(A))) = (A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A))) \cup (A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)))\}$.

4. $D(p, ps) = \{A \subset X : A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)) = A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)))\}$.

5. $D(p, b) = \{A \subset X : A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)) = (A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A))) \cup (A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)))\}$.

6. $D(s, ps) = \{A \subset X : A \cap \tau_{\gamma\vee\gamma'}\text{-}(_{\gamma\vee\gamma'}(A)) = A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A))\}$.

7. $D(s,b) = \{A \subset X : A \cap \tau_{\gamma\vee\gamma'}\text{-}(_{\gamma\vee\gamma'}(A)) = (A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A))) \cup (A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)))\}$.

8. $D(b,sp) = \{A \subset X : (A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A))) \cup (A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A))) = A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A))\}$.

Example 3.2. *Let $X = \{a,b,c\}$ and $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a,c\}, \{a,b\}\}$. We define the operations $\gamma, \gamma' : \tau \to \wp(X)$ as follows*

$$A^{\gamma} = A^{\gamma'} = \begin{cases} A & \text{if } A = \{a\} \text{ or } \{c\}, \\ A \cup \{a,c\} & \text{if } A \neq \{a\} \text{ and } \{c\} \end{cases}$$

Observe that:

1. $\tau_{\gamma\vee\gamma'} = \{\emptyset, X, \{a\}, \{c\}, \{a,c\}\}$
2. $D(\alpha,s) = \{\emptyset, X, \{b\}, \{c\}, \{a,c\}\}$
3. $D(\alpha,sp) = \{\emptyset, X, \{b\}, \{a,c\}\}$
4. $D(\alpha,b) = \{\emptyset, X, \{b\}, \{a,c\}\}$.
5. $D(p,ps) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,c\}\}$.
6. $D(p,b) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,c\}\}$.
7. $D(s,ps) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,c\}\}$.
8. $D(s,b) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$.
9. $D(b,sp) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$.
10. The $\gamma \vee \gamma'$-semi open set $= \{\emptyset, X, \{a\}, \{c\}, \{a,b\}, \{b,c\}\}$.
11. The $\gamma \vee \gamma'$-semi preopen set $=\{\emptyset, X, \{a\}, \{c\}, \{a,b\}, \{b,c\}\}$.
12. The $\gamma \vee \gamma'$-b- open set $= \{\emptyset, X, \{a\}, \{c\}, \{a,b\}, \{b,c\}\}$.

Example 3.3. *Let $X = \{a,b,c\}$ and $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a,c\}, \{a,b\}\}$. We define the operations $\gamma, \gamma' : \tau \to \wp(X)$ as follows*

$$A^{\gamma} = \begin{cases} A & \text{if } A = \{a\}, \\ A \cup \{a,c\} & \text{if } A \neq \{a\} \end{cases}$$

$$A^{\gamma'} = \begin{cases} int(cl(A)) & \text{if } A = \{a\}, \\ X & \text{if } A \neq \{a\} \end{cases}$$

Observe that:

1. $\tau_{\gamma\vee\gamma'} = \{\emptyset, X\}$.
2. $D(\alpha, s) = \wp(X)$
3. $D(\alpha, sp) = \{\emptyset, X\}$
4. $D(\alpha, b) = \wp(X)$.
5. $D(p, ps) = \wp(X)$.
6. $D(p, b) = \wp(X)$.
7. $D(s, ps) = \{\emptyset, X\}$.
8. $D(s, b) = \{\emptyset, X\}$.
9. $D(b, sp) = \{\emptyset, X\}$.
10. *The* $\gamma \vee \gamma'$*-semi open set* $= \{\emptyset, X\}$.
11. *The* $\gamma \vee \gamma'$*-semi preopen set* $=\wp(X)$.
12. *The* $\gamma \vee \gamma'$*-b- open set* $= \wp(X)$.

Theorem 3.4. *The following statements hold for a topological space* (X, τ) *with the operations* γ *and* γ'*:*

1. *Every* $D(\alpha, sp)$*-set is* $D(p, sp)$*-set.*
2. *Every* $D(\alpha, sp)$*-set is* $D(s, sp)$*-set.*
3. *Every* $D(p, sp)$*-set is* $D(b, sp)$*-set.*
4. *Every* $D(s, sp)$*-set is* $D(b, sp)$*-set.*

Proof. (1). Let A be a $D(\alpha, sp)$-set. Then $A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A))) = A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)))$. Now, $A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A))) = A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A))) \subset A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)) \subset A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)))$. Thus it follows that A is a $D(p, sp)$-set.
(2). Let A be a $D(\alpha, sp)$-set. Then $A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A))) = A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)))$. Now, $A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A))) = A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A))) \subset A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)) \subset A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)) \subset A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)))$. Thus it follows that A is a $D(s, sp)$-set.
(3). Let A be a $D(p, sp)$-set. Then $A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)) = A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)))$. Since $A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A))) = A \cap \tau_{\gamma\vee\gamma'}\text{-}(\tau_{\gamma\vee\gamma'}\text{-}(A)) \subset$

$A \cap \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)) \cup A \cap \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)) = A \cap [\tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)) \cup \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))] \subset A \cap [\tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)) \cup \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))] \subset A \cap \cup \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)))[\tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))] = A \cap \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)))$. It follows that A is a $D(p, sp)$-set.
(4) Analogous to (3). □

Example 3.5. *In Example 3.2, we can see that the converse of the Theorem 3.4, are not necessarily true.*

Theorem 3.6. *The following statements hold for a topological space (X, τ) with the operations γ and γ':*

1. *A is $\gamma \vee \gamma'$-α-open if, and only if it is both $\gamma \vee \gamma'$-semiopen and $D(\alpha, s)$-set;*
2. *A is $\gamma \vee \gamma'$-α-open if, and only if it is both $\gamma \vee \gamma'$-sp-open and $D(\alpha, sp)$-set;*
3. *A is $\gamma \vee \gamma'$-preopen if, and only if it is both $\gamma \vee \gamma'$-b-open and $D(p, sp)$-set;*
4. *A is $\gamma \vee \gamma'$-preopen if, and only if it is both $\gamma \vee \gamma'$-sp-open and $D(p, sp)$-set;*
5. *A is $\gamma \vee \gamma'$-semiopen if, and only if it is both $\gamma \vee \gamma'$-sp-open and $D(\alpha, p)$-set;*
6. *A is $\gamma \vee \gamma'$-semiopen if, and only if it is both $\gamma \vee \gamma'$-b-open and $D(s, b)$-set;*
7. *A is $\gamma \vee \gamma'$-semiopen if, and only if it is both $\gamma \vee \gamma'$-sp-open and $D(s, sp)$-set;*
8. *A is $\gamma \vee \gamma'$-b-open if, and only if it is both $\gamma \vee \gamma'$-sp-open and $D(b, sp)$-set.*

Proof. (1). By Definition 3.1, it is obvious that every $\gamma \vee \gamma'$-α-open set is $D(\alpha, s)$-set. For the sufficiency of (1); Suppose that A is both $\gamma \vee \gamma'$-semiopen and $D(\alpha, s)$-set. Then $A \subset \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))$ and $A \cap \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))) = A \cap \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))$. Since $A \subset \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))$. It follows that $A \cap \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$\tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))) = A$. Thus A is $\gamma \vee \gamma'$-α-open.
(2). Analogous to (1).

(3). Let A be a $\gamma \vee \gamma'$-preopen set. Then $A \subset \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))$. Hence $A \subset \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))[\tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))$. Therefore A is $\gamma \vee \gamma'$-b-open. On the other hand by Definition 3.1 it follows that every $\gamma \vee \gamma'$-preopen set is $D(p, b)$-set. For the sufficiency of (3); Suppose that A is both $\gamma \vee \gamma'$-b-open and $D(p, b)$-set. Then $A \subset \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)) \cap \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))$ and $A \cap \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)) = A \cap \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)) \cap A \cap \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)) = A \cap [\tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))[\tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))]$. Since $A \subset \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)) \cap \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))$, $A \cap \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)) = A$, showing that A is $\gamma \vee \gamma'$-preopen.
(4). Let A be a $\gamma \vee \gamma'$-preopen set. Then $A \subset \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))$. Hence $A \subset \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)))$. Therefore A is $\gamma \vee \gamma'$-sp-open. On the other hand by Definition 3.1 it follows that every $\gamma \vee \gamma'$-preopen set is $D(p, sp)$-set. Sufficiency of (4) is analogous to sufficiency of (3).
(5). Let A be a $\gamma \vee \gamma'$-semiopen set. Then $A \subset \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))$. Then we have $A \subset\subset \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)))$. Therefore A is $\gamma \vee \gamma'$-sp-open. On the other hand by Definition 3.1 it follows that every $\gamma \vee \gamma'$-semiopen set is $\gamma \vee \gamma'$-$D(\alpha, p)$-set. Sufficiency of (4) is analogous to sufficiency of (3).
(6) Analogous to (3).
(7). Let A be a $\gamma \vee \gamma'$-semiopen set. Then $A \subset \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))$. Then we have $A \cap \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)) \subset\subset \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)))$. Therefore A is $\gamma \vee \gamma'$-sp-open. On the other hand by Definition 3.1 it follows that every $\gamma \vee \gamma'$-semiopen set is $\gamma \vee \gamma'$-$D(s, sp)$-set. Sufficiency of (7) is analogous to sufficiency of (1).
(8). Let A be a $\gamma \vee \gamma'$-b-open set then $A \subset \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)) \cup \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))$ implies that $A \subset \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))$ or $A \subset \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))$. If $A \subset \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A))$ then $A \subset \tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(\tau_{\gamma\vee\gamma'}$-$(A)))$, showing that A is $\gamma \vee \gamma'$-sp-open. On the other hand by Definition 3.1 it follows that every $\gamma \vee \gamma'$-b-open set is $\gamma \vee \gamma'$-$D(b, sp)$-set. Sufficiency of (8) is analogous to sufficiency of (1). □

Remark 3.7. *In a topological space (X, τ), the following hold:*

1. *The notions of $\gamma \vee \gamma'$-semi open set and $D(\alpha, s)$-sets are independent.*
2. *The notions of $\gamma \vee \gamma'$-sp-open and $D(\alpha, sp)$-sets are independent.*
3. *The notions of $\gamma \vee \gamma'$-b-open and $D(p, b)$-sets are independent.*
4. *The notions of $\gamma \vee \gamma'$-sp-open and $D(p, sp)$-sets are independent.*
5. *The notions of $\gamma \vee \gamma'$-sp-open and $D(\alpha, p)$-sets are independent.*
6. *The notions of $\gamma \vee \gamma'$-b-open and $D(s, b)$-sets are independent.*

7. *The notions of* $\gamma \vee \gamma'$*-sp-open and* $D(s, sp)$*-sets are independent.*

8. *The notions of* $\gamma \vee \gamma'$*-sp-open and* $D(b, sp)$*-sets are independent.*

Example 3.8. *In Examples 3.2, and 3.3, we can obtain all needed information related with Remark 3.7.*

4. Some decomposition theorems

Definition 4.1. *Let* (X, τ) *and* (Y, σ) *be two topological spaces and let* $\gamma, \gamma' : \tau \longrightarrow \wp(X)$ *be operations on* τ *and* $\beta, \beta' : \sigma \longrightarrow \wp(X)$ *be operations on* σ*. A function* $f : (X, \tau) \longrightarrow (Y, \sigma)$ *is said to be:*

1. $(\gamma \vee \gamma', \beta \vee \beta')$*-precontinuous if for each* $x \in X$ *and each* $\beta \vee \beta'$*-open set* V *of* Y*, there exist a* $\gamma \vee \gamma'$*-open set* U *of* X *such that* $f(U) \subseteq V$*.*

2. $(\gamma \vee \gamma', \beta \vee \beta')$*-b-continuous if for each* $x \in X$ *and each* $\beta \vee \beta'$*-open set* V *of* Y*, there exist a* $\gamma \vee \gamma'$*-b-open set* U *of* X *such that* $f(U) \subseteq V$*.*

3. $(\gamma \vee \gamma', \beta \vee \beta')$*-*$\alpha$*-continuous if for each* $x \in X$ *and each* $\beta \vee \beta'$*-open set* V *of* Y*, there exist a* $\gamma \vee \gamma'$*-*α*-open set* U *of* X *such that* $f(U) \subseteq V$*.*

4. $(\gamma \vee \gamma', \beta \vee \beta')$*-semicontinuous if for each* $x \in X$ *and each* $\beta \vee \beta')$*-open set* V *of* Y*, there exist a* $\gamma \vee \gamma'$*-semiopen* U *set of* X *such that* $f(U) \subseteq V$*..*

5. $(\gamma \vee \gamma', \beta \vee \beta')$*-sp-continuous if for each* $x \in X$ *and each* $\beta \vee \beta')$*-open set* V *of* Y*, there exist a* $\gamma \vee \gamma'$*-sp-open set* U *of* X *such that* $f(U) \subseteq V$*..*

6. $(\gamma \vee \gamma', \beta \vee \beta')$*-*$D(\alpha, s)$*-continuous if* $f^{-1}(V) \in \gamma \vee \gamma'$*-*$D(\alpha, s)$ *for each* $V \in \sigma_{\beta \vee \beta'}$*.*

7. $(\gamma \vee \gamma', \beta \vee \beta')$*-*$D(\alpha, p)$*-continuous if* $f^{-1}(V) \in \gamma \vee \gamma'$*-*$D(\alpha, p)$ *for each* $V \in \sigma_{\beta \vee \beta'}$*.*

8. $(\gamma \vee \gamma', \beta \vee \beta')$*-*$D(\alpha, sp)$*-continuous if* $f^{-1}(V) \in \gamma \vee \gamma'$*-*$D(\alpha, sp)$ *for each* $V \in \sigma_{\beta \vee \beta'}$*.*

9. $(\gamma \vee \gamma', \beta \vee \beta')$*-*$D(p, b)$*-continuous if* $f^{-1}(V) \in \gamma \vee \gamma'$*-*$D(p, b)$ *for each* $V \in \sigma_{\beta \vee \beta'}$*.*

10. $(\gamma \vee \gamma', \beta \vee \beta')$*-*$D(p, sp)$*-continuous if* $f^{-1}(V) \in \gamma \vee \gamma'$*-*$D(p, sp)$ *for each* $V \in \sigma_{\beta \vee \beta'}$*.*

11. $(\gamma \vee \gamma', \beta \vee \beta')$-$D(s, b)$-*continuous if* $f^{-1}(V) \in \gamma \vee \gamma'$-$D(s, b)$ *for each* $V \in \sigma_{\beta \vee \beta'}$.

12. $(\gamma \vee \gamma', \beta \vee \beta')$-$D(s, sp)$-*continuous if* $f^{-1}(V) \in \gamma \vee \gamma'$-$D(s, sp)$ *for each* $V \in \sigma_{\beta \vee \beta'}$.

13. $(\gamma \vee \gamma', \beta \vee \beta')$-$D(b, sp)$-*continuous if* $f^{-1}(V) \in \gamma \vee \gamma'$-$D(b, sp)$ *for each* $V \in \sigma_{\beta \vee \beta'}$.

Remark 4.2. *It is easy to see, from Theorem 3.6, we can obtain many relations between the diffrent forms of continuity described in Definition 4.1, for example:*

1. *Every* $(\gamma \vee \gamma', \beta \vee \beta')$-$D(\alpha, sp)$-*continuous is* $(\gamma \vee \gamma', \beta \vee \beta')$-$D(p, sp)$-*continuous.*

2. *Every* $(\gamma \vee \gamma', \beta \vee \beta')$-$D(\alpha, sp)$-*continuous is* $(\gamma \vee \gamma', \beta \vee \beta')$-$D(s, sp)$-*continuous.*

3. *Every* $(\gamma \vee \gamma', \beta \vee \beta')$-$D(p, ps)$-*continuous is* $(\gamma \vee \gamma', \beta \vee \beta')$-$D(p, sp)$-*continuous.*

4. *Every* $(\gamma \vee \gamma', \beta \vee \beta')$-$D(s, sp)$-*continuous is* $(\gamma \vee \gamma', \beta \vee \beta')$-$D(b, sp)$-*continuous.*

As an immediate consequence of Theorem 3.6,we have the followings theorems.

Theorem 4.3. *For a function* $f : (X, \tau) \rightarrow (Y, \sigma)$, *the following statements are equivalent:*

1. f *is* $(\gamma \vee \gamma', \beta \vee \beta')$-$\alpha$-*continuous,*

2. f *is* $(\gamma \vee \gamma', \beta \vee \beta')$-*semicontinuous and* $(\gamma \vee \gamma', \beta \vee \beta')$-$D(\alpha, s)$-*continuous,*

3. f *is* $(\gamma \vee \gamma', \beta \vee \beta')$-*b-continuous and* $(\gamma \vee \gamma', \beta \vee \beta')$-$D(\alpha, sp)$-*continuous.*

Theorem 4.4. *For a function* $f : (X, \tau) \rightarrow (Y, \sigma)$, *the following statements are equivalent:*

1. f *is* $(\gamma \vee \gamma', \beta \vee \beta')$-*precontinuous,*

2. f *is* $(\gamma \vee \gamma', \beta \vee \beta')$-*b-continuous and* $(\gamma \vee \gamma', \beta \vee \beta')$-$D(p, b)$-*continuous,*

3. f *is* $(\gamma \vee \gamma', \beta \vee \beta')$*-b-continuous and* $(\gamma \vee \gamma', \beta \vee \beta')$*-*$D(p, sp)$*-continuous.*

Theorem 4.5. *For a function* $f : (X, \tau) \to (Y, \sigma)$*, the following statements are equivalent:*

1. f *is* $(\gamma \vee \gamma', \beta \vee \beta')$*-semicontinuous,*
2. f *is* $(\gamma \vee \gamma', \beta \vee \beta')$*-b-continuous and* $(\gamma \vee \gamma', \beta \vee \beta')$*-*$D(s, b)$*-continuous,*
3. f *is* $(\gamma \vee \gamma', \beta \vee \beta')$*-sp-continuous and* $(\gamma \vee \gamma', \beta \vee \beta')$*-*$D(\alpha, p)$*-continuous,*
4. f *is* $(\gamma \vee \gamma', \beta \vee \beta')$*-sp-continuous and* $(\gamma \vee \gamma', \beta \vee \beta')$*-*$D(s, sp)$*-continuous.*

Theorem 4.6. *For a function* $f : (X, \tau) \to (Y, \sigma)$*, the following statements are equivalent:*

1. f *is* $(\gamma \vee \gamma', \beta \vee \beta')$*-semicontinuous,*
2. f *is* $(\gamma \vee \gamma', \beta \vee \beta')$*-b-continuous and* $(\gamma \vee \gamma', \beta \vee \beta')$*-*$D(b, sp)$*-continuous.*

References

[1] S. Kasahara, “Operation-compact spaces”, *Mathematics japonica*, vol. 24, pp. 97-105, 1979.

[2] R. Nirmala and N. Rajesh, “Bioperation-regular open sets”, *Aryabhata journal mathematics informatics*, vol. 8, no. 1, pp. 53-62, 2016.

[3] R. Nirmala and N. Rajesh, “Generalization of semiopen sets via bioperation”(under preparation).

[4] H. Ogata and H. Maki, “Bioperation on topological spaces”, *Mathematics japonica,* vol. 38, no. 5, pp. 981-985, 1993.

[5] M. Perzemshi, “A decomposition of continuity and α-continuity”, *Acta mathematica hungarica*, vol. 61, no. 1/2), pp. 93-98, 1993, doi: 10.1007/BF01872101