Acta Univ. Sapientiae, Mathematica, **10**, 2 (2018) 319–328

DOI: 10.2478/ausm-2018-0024

Θ-modifications on weak spaces

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Abstract. In this article, we want to study and investigate if it is possible to use the notions of weak structures to develop a new theory of θ - modifications in weak spaces and study their properties, finally we study some forms of weak continuity using this modifications.

1 Introduction

In [5], Császar and Makai Jr. introduced and studied the notions of $\delta_{\mu_1\mu_2}$-open sets and $\theta_{\mu_1\mu_2}$-open sets defined by two generalized topologies μ_1 and μ_2 on a nonempty set X and they proved that: $\delta_{\mu_1\mu_2}$ and $\theta_{\mu_1\mu_2}$ are generalized topologies on X and $\theta_{\mu_1\mu_2} \subseteq \delta_{\mu_1\mu_2} \subseteq \mu_1$. The notions of $(\theta_{w_1w_2}, \theta_{\sigma_1\sigma_2})$-continuous was introduced and characterized by W. K. Min in [6], also introduced and characterized the notions of $(\delta_{w_1w_2}, \delta_{\sigma_1\sigma_2})$-continuous on generalized topological

2010 Mathematics Subject Classification: 54C05, 54C08, 54C10
Key words and phrases: weak structure, w-space, w-closure, $(\Theta_{w_1w_2}, \Theta_{\nu_1\nu_2})$-continuous functions

spaces and $(\delta_{w_1w_2}, \theta_{\nu_1\nu_2})$-continuous. W. K. Min in [7], introduced the notions of mixed weak $(\mu, \nu_1\nu_2)$-continuity between a generalized topology μ and two generalized topologies ν_1, ν_2, also he introduced and characterized continuity in terms of mixed generalized $(\nu_1, \nu_2)'$-semiopen sets, $(\nu_1, \nu_2)'$-preopen sets, (ν_1, ν_2)-preopen sets [4] ,(ν_1, ν_2)'-β'-open sets and $\theta(\nu_1, \nu_2)$'-open sets [5] . Ugur Sengul in [12], using the δ and θ-modifications in bigeneralized topologies, introduced the notion of $(\delta_{\mu_1\mu_2}, \theta_{\sigma_1\sigma_2})$-continuity between two Bi-GTSś. Also he characterized such continuity in terms of mixed generalized open sets: $\delta_{\mu_1\mu_2}$-open sets, $\theta_{\mu_1\mu_2}$-open sets. In this article, we want to study if it is possible, using weak structures to make a new theory related to θ -modifications of weak spaces and study some weak forms of continuity.

2 Preliminaries

Definition 1 [9] *Let X be a nonempty set. A subfamily w_X of the power set $P(X)$ is called a weak structure on X if it satisfies the following:*

1. $\emptyset \in w_X$ *and* $X \in w_X$.
2. *For* $U_1, U_2 \in w_X$, $U_1 \cap U_2 \in w_X$

The pair (X, w_X) is called a w-space on X. An element $U \in w_X$ is called w-open set and the complement of a w-open set is a w-closed set

Definition 2 [9] *Let (X, w_X) be a w-space. For a subset A of X,*

1. *The w-closure of A is defined as* $wC(A) = \bigcap\{F : A \subseteq F, X \setminus F \in w_X\}$.
2. *The w-interior of A is defined as* $wI(A) = \bigcup\{U : U \subseteq A, U \in w_X\}$.

Theorem 1 [9] *Let (X, w_X) be a w-space on X and A, B subsets of X. Then the following hold:*

1. *If* $A \subseteq B$, *then* $wI(A) \subseteq wI(B)$ *and* $wC(A) \subseteq wC(B)$.
2. $wI(wI(A)) = wI(A)$ *and* $wC(wC(A)) = wC(A)$.
3. $wC(X \setminus A) = X \setminus wI(A)$ *and* $wI(X \setminus A) = X \setminus wC(A)$.
4. $x \in wC(A)$ *if and only if* $U \cap A \neq \emptyset$, *for all* $U \in w_X$ *with* $x \in U$.
5. $x \in wI(A)$ *if and only if there exists* $U \in w_X$ *with* $x \in U$, *such that* $U \subseteq A$.

6. *If* A *is* w*-closed (resp.* w*-open), then* $wC(A) = A$ *(resp.* $wI(A) = A$*).*

Theorem 2 [11] *Let* (X, w_X) *be a* w*-space on* X *and* A, B *subsets of* X*. Then the following hold:*

1. $wI(A \cap B) = wI(A) \cap wI(B)$.

2. $wC(A \cup B) = wC(A) \cup wC(B)$.

Theorem 3 *Let* (X, w_X) *be a* w*-space on* X *and* A, B *subsets of* X*. Then the following hold:*

1. $wI(A) \cup wI(B) \subseteq wI(A \cup B)$.

2. $wC(A \cap B) \subseteq wC(A) \cap wC(B)$.

3 Modification on weak structures

Throughout this paper if w_1, w_2 are two weak structures on a nonempty set X. Then (X, w_1, w_2) is called a biweak space. Recall that Császar, A. [3], showed that the δ and θ-modifications of topological spaces can be generalized for the case when the topology is replaced by the generalized topologies μ_1, μ_2 in the sense of [1]. W. K. Min [6], gave a characterization for $(\theta_{\mu_1\mu_2}, \theta_{\sigma_1\sigma_2})$-continuity and introduce the concepts of $(\delta_{\mu_1\mu_2}, \delta_{\sigma_1\sigma_2})$-continuity on generalized topological spaces and investigate the relationship between $(\delta_{\mu_1\mu_2}, \theta_{\sigma_1\sigma_2})$-continuity, $(\theta_{\mu_1\mu_2}, \theta_{\sigma_1\sigma_2})$-continuity and $(\delta_{\mu_1\mu_2}, \delta_{\sigma_1\sigma_2})$-continuity. In our case, we want to study what happen when the generalized topologies are replaced by weak structures.

Definition 3 *Let* (X, w_1, w_2) *be a biweak space. A subset* A *of* X *is said to be* $\Upsilon_{w_1w_2}$*-open (resp.* $\Upsilon_{w_1w_2}$*-closed) if* $A = w_1I(w_2C(A))$ *(resp.* $A = w_1C(w_2I(A))$*).*

Example 1 *Let* (X, w_1, w_2) *be a biweak space, where* $X = \{a, b, c\}$*,* $w_1 = \{\emptyset, X, \{a\}, \{b\}\}$ *and* $w_2 = \{\emptyset, X, \{a\}, \{c\}\}$*.*
Observe that the set $A = \{b\}$ *is* $\Upsilon_{w_1w_2}$*-open, the set* $B = \{c\}$ *is* $\Upsilon_{w_2w_1}$*-open and the set* $C = \{a, b\}$ *is not* $\Upsilon_{w_2w_1}$*-open set.*

Definition 4 *Let* (X, w_1, w_2) *be a biweak space.*

1. $A \in \theta_{w_1w_2}$ *if and only if for each* $x \in A$*, there exists an* $U \in w_1$ *such that* $x \in U \subseteq w_2C(U) \subseteq A$*.*

2. $A \in \delta_{w_1 w_2}$ *if and only if* $A \subset X$ *and if* $x \in A$*, there exists a* w_2*-closed set* F *such that* $x \in w_1 I(F) \subseteq A$.

Example 2 *In Example 1:*

1. $\theta_{w_1 w_2} = \{\emptyset, X, \{b\}, \{a, b\}\}$,
2. $\delta_{w_1 w_2} = \{\emptyset, X, \{b\}, \{a, b\}\}$,
3. $\theta_{w_2 w_1} = \{\emptyset, X, \{c\}, \{a, c\}\}$,
4. $\delta_{w_2 w_1} = \{\emptyset, X, \{a, c\}, \{c\}\}$.

Example 3 *Let* (X, w_1, w_2) *be a biweak space, where* $X = \{a, b, c\}$*,* $w_1 = \{\emptyset, X, \{a\}, \{b\}\}$ *and* $w_2 = \{\emptyset, X, \{a\}, \{a, b\}, \{c\}\}$*.*
Observe that:

1. $\theta_{w_1 w_2} = \{\emptyset, X, \{b\}, \{a, b\}\}$,
2. $\delta_{w_1 w_2} = \{\emptyset, X, \{a, b\}, \{b\}\}$,
3. $\theta_{w_2 w_1} = \{\emptyset, X, \{c\}, \{a, c\}\}$,
4. $\delta_{w_2 w_1} = \{\emptyset, X, \{a, c\}, \{c\}\}$.

Example 4 *Let* (X, w_1, w_2) *be a biweak space, where* $X = \{a, b, c\}$*,* $w_1 = \{\emptyset, X, \{a\}, \{b\}, \{c\}\}$ *and* $w_2 = \{\emptyset, X, \{a\}, \{a, b\}, \{c\}\}$*. Observe that:*

1. $\theta_{w_1 w_2} = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{a, b\}\}$,
2. $\delta_{w_1 w_2} = \{\emptyset, X, \{c\}, \{a, b\}, \{b, c\}\}$,
3. $\theta_{w_2 w_1} = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$,
4. $\delta_{w_2 w_1} = \{\emptyset, X, \{a, b\}, \{c\}\}$.

Example 5 *Let* $X = \{a, b, c\}$ *with weak structures* $w_1 = \{\emptyset, X, \{b\}\}$ *and* $w_2 = \{\emptyset, X, \{a\}\}$*. Observe that:*

1. $\theta_{w_1 w_2} = \{\emptyset, X\}$,
2. $\delta_{w_1 w_2} = \{\emptyset, X, \{b\}\}$,
3. $\theta_{w_2 w_1} = \{\emptyset, X\}$,

4. $\delta_{w_2 w_1} = \{\emptyset, X, \{a\}\}$.

Remark 1 *According with Example 4, $\delta_{w_1 w_2}$ is not necessary a weak structures on X, then first of all, we have an answer. We can not doing similarly modification as [5], if we replace generalized topology by weak structure.*

Theorem 4 *Let (X, w_1, w_2) be a biweak space. The collection $\theta_{w_1 w_2}$ is a strong generalized topology on X.*

Proof. It is easy to see that: $\emptyset$ and X belong to $\theta_{w_1 w_2}$. Now consider $\{U_i : i \in I\}$ a collection of elements of $\theta_{w_1 w_2}$ and $x \in \bigcup_{i \in I} U_i$, then for some $i \in I$, $x \in U_i$ and then there is $V_i \in w_1$, such that $x \in U_i \subseteq w_2C(V_i) \subseteq U_i \subseteq \bigcup_{i \in I} U_i$. It follows that $\bigcup_{i \in I} U_i \in \theta_{w_1 w_2}$. □

Theorem 5 *Let (X, w_1, w_2) be a biweak space. The collection $\theta_{w_1 w_2}$ is a weak structure on X.*

Proof. It is easy to see that: $\emptyset$ and X belong to $\theta_{w_1 w_2}$. Now consider U_1, U_2 two elements of $\theta_{w_1 w_2}$ and $x \in U_1 \cap U_2$, then $x \in U_i$ for $i = 1, 2$. Then there exists $V_i \in w_i$ for $i = 1, 2$, such that $x \in V_i$ and $w_2C(V_i) \subseteq U_i$. It follows that $x \in V_1 \cap V_2$ and $w_2C(V_1) \cap w_2C(V_2) \subseteq U_1 \cap U_2$. But $V_1 \cap V_2 \in w_1$ and $V_1 \cap V_2 \subseteq w_2C(V_1 \cap V_2) \subseteq w_2C(V_1) \cap w_2C(V_2) \subseteq U_1 \cap U_2$. Hence $U_1 \cap U_2 \in \theta_{w_1 w_2}$. □

Remark 2 *Observe that if (X, w_1, w_2) is a biweak space, $\theta_{w_1 w_2}$ is a topology on X.*

Theorem 6 *Let (X, w_1, w_2) be a biweak space. The collection $\delta_{w_1 w_2}$ is a strong generalized topology on X.*

Proof. It is easy to see that: $\emptyset$ and X belong to $\delta_{w_1 w_2}$. Consider $\{V_i : i \in I\}$ a collection of elements of $\delta_{w_1 w_2}$ and $x \in \bigcup_{i \in I} V_i$, then for some $i \in I$, $x \in V_i$ and then there is w_2-closed set F such that $x \in w_1I(F) \subseteq V_i$ and hence, $x \in w_1I(F) \subseteq V_i \subseteq \bigcup_{i \in I} V_i$. In consequence, $\bigcup_{i \in I} V_i \in \delta_{w_1 w_2}$. □

Remark 3 *According with Example 3, $\theta_{w_1 w_2} \subsetneq w_1$ and by Example 4, $\theta_{w_1 w_2} \subsetneq \delta_{w_1 w_2}$ and $\delta_{w_1 w_2} \subsetneq w_1$.*

Remark 4 *Let (X, w_1, w_2) be a biweak space. There are no relation between $\theta_{w_1 w_2}$ and $\delta_{w_1 w_2}$, see Examples 4 and 5.*

Remark 5 *If we start with a biweak space* (X, w_1, w_2). *We obtain that* $\theta_{w_1w_2}$ *is a topology on* X, *see Remark 2.* $\delta_{w_1w_2}$ *is a strong generalized topology on* X, *see Theorem 6 and there are no relation between* $\theta_{w_1w_2}$ *and* $\delta_{w_1w_2}$, *see Examples 4 and 5.*

Definition 5 $A \in \theta_{w_1w_2}$ *is called* $\theta_{w_1w_2}$*-open set and its complement is called* $\theta_{w_1w_2}$*-closed.*

According with Definition 5, we define the $\theta_{w_1w_2}$-closure of a subset A of X, as follows:

Definition 6 *Let* (X, w_1, w_2) *be a biweak space.*

1. *The* $\theta_{w_1w_2}$*-closure of* A *is defined as:*
   $C\theta_{w_1w_2}(A) = \bigcap\{F : A \subseteq F, F$ *is* $\theta_{w_1w_2}$*-closed set in* $X\}$.

2. *The* $\theta_{w_1w_2}$*-interior of* A *is defined as:*
   $I\theta_{w_1w_2}(A) = \bigcup\{U : U \subseteq A, U$ *is* $\theta_{w_1w_2}$*-open set in* $X\}$.

3. $\gamma\theta_{w_1w_2}(A) = \{x \in X : w_2C(U) \cap A \neq \emptyset,$ *for every* $U \in w_1$ *containing* $x\}$.

Example 6 *In Example 2. The* $C\theta_{w_1w_2}(\emptyset) = \emptyset$, $C\theta_{w_1w_2}(X) = X$, $C\theta_{w_1w_2}(\{a\}) = \{a, c\}$, $C\theta_{w_1w_2}(\{b\}) = X$, $C\theta_{w_1w_2}(\{c\}) = \{c\}$, $C\theta_{w_1w_2}(\{a, b\}) = X$, $C\theta_{w_1w_2}(\{a, c\}) = \{a, c\}$, $C\theta_{w_1w_2}(\{b, c\}) = X$.

Theorem 7 *Let* (X, w_1, w_2) *and* (X, v_1, v_2) *be two biweak space and* $A \subseteq X$. *If* $w_1 \subseteq v_1$ *and* $w_2 \subseteq v_2$. *Then* $\theta_{w_1w_2} \subseteq \theta_{v_1v_2}$

Proof. Let $A \in \theta_{w_1w_2}$ and $x \in A$, then there exists an $U \in w_1$ such that $x \in U \subseteq w_2C(U) \subseteq A$. Since $w_1 \subseteq v_1$, $U \in v_1$ and $v_2C(U) \subseteq w_2C(U) \subseteq A$. □

Theorem 8 *Let* (X, w_1, w_2) *be a biweak space and* $A \subseteq X$. *The following are true:*

1. $A \subseteq \gamma\theta_{w_1w_2}(A) \subseteq C\theta_{w_1w_2}(A)$.

2. A *is* $\theta_{w_1w_2}$*-closed if and only if* $A = \gamma\theta_{w_1w_2}(A)$.

3. $x \in I\theta_{w_1w_2}(A)$ *if and only if there exists a* w_1*-open set* U *containing* x *such that* $x \in U \subseteq w_2C(U) \subseteq A$.

4. *if* A *is* w_2*-open, then* $w_1C(A) = \gamma\theta_{w_1w_2}(A)$.

Proof. 1. Since $\theta_{w_1w_2}$ is a weak space, the result follows.
2. If A is $\theta_{w_1w_2}$-closed, then $A = C\theta_{w_1w_2}(A)$, Now using 1, the result follows.
3. Is a consequence of Definition 6.
4. Let $x \in w_1C(A)$ and U any w_1-open set containing x, then $U \cap A \neq \emptyset$, follows that $w_2C(U) \cap A \neq \emptyset$ and then $w_1C(A) \subseteq \gamma\theta_{w_1w_2}(A)$. Now consider $x \in \gamma\theta_{w_1w_2}(A)$, then for each w_1-open set U containing x, $w_2C(U) \cap A \neq \emptyset$, and then there exists an element $z \in w_2C(U) \cap A$, since A is w_2-open, $z \in A$, therefore $x \in w_1C(A)$. □

4 Modification on weak continuous functions

Definition 7 *Let* (X, w_1, w_2) *and* (Y, v_1, v_2) *be two biweak spaces. A function* $f : X \to Y$ *is said to be* $(\Theta_{w_1w_2}, \Theta_{v_1v_2})$*-continuous if for every* $\Theta_{v_1v_2}$*-open set* V*,* $f^{-1}(V)$ *is* $\Theta_{w_1w_2}$*-open.*

Observe that if (X, w_1, w_2) and (Y, v_1, v_2) are two biweak spaces, $\Theta_{w_1w_2}$ and $\Theta_{v_1v_2}$ are topologies, then the notion of $(\Theta_{w_1w_2}, \Theta_{v_1v_2})$-continuous functions is similar to the well known concept of continuous functions.

Example 7 *In Example 4. Observe that:*

1. $\theta_{w_1w_2} = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{a, b\}\}$,
2. $\theta_{w_2w_1} = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\{b, c\}\}$,

The identity function $f : X \to X$ *is* $(\Theta_{w_2w_1}, \Theta_{w_1w_2})$*-continuous but is not* $(\Theta_{w_1w_2}, \Theta_{w_2w_1})$*-continuous.*

Theorem 9 *Let* (X, w_1, w_2) *and* (Y, v_1, v_2) *be two biweak spaces; let* $f : X \to Y$*. Then the following are equivalent:*

1. f *is* $(\Theta_{w_1w_2}, \Theta_{v_1v_2})$*-continuous,*
2. *For each* $x \in X$ *and each* $\Theta_{v_1v_2}$*-open set* V *containing* $f(x)$*, there exists a* $\Theta_{w_1w_2}$*-open set* U *containing* x*such that* $f(U) \subseteq V$*.*
3. *For each* $x \in X$ *and each* $\Theta_{v_1v_2}$*-open set* V *containing* $f(x)$*, there exists a* w_1*-open set* U *containing* x *such that* $f(w_2C(U) \subseteq V$*.*

Proof. The proof follows applying definition. □

Definition 8 *Let* (X, w_1) *be a weak space and* (Y, v_1, v_2) *be a biweak space. A function* $f : (X, w_1) \to (Y, v_1, v_2)$ *is said to be faintly* $(w_1, \Theta_{v_1 v_2})$*-continuous if for every* $\Theta_{v_1 v_2}$*-open set* U*,* $f^{-1}(U)$ *is* w_1*-open.*

Example 8 *Let* (X, w_1, w_2) *and* (Y, v_1, v_2) *be a biweak spaces, where* $X = Y = \{a, b, c\}$*,* $w_1 = \{\emptyset, X, \{a\}, \{b\}, \{c\}\}$*,* $w_2 = \{\emptyset, X, \{a\}, \{a, b\}, \{c\}\}$*,* $v_1 = \{\emptyset, Y, \{a\}, \{b\}\}$ *and* $v_2 = \{\emptyset, Y, \{a\}, \{c\}\}$*.*
Observe that:

1. $\theta_{w_1 w_2} = \{\emptyset, X, \{b\}, \{a, b\}\}$,
2. $\theta_{w_2 w_1} = \{\emptyset, X, \{c\}, \{a, c\}\}$,
3. $\theta_{v_1 v_2} = \{\emptyset, X, \{b\}, \{a, b\}\}$,
4. $\theta_{v_2 v_1} = \{\emptyset, X, \{c\}, \{a, c\}\}$.

Consider a function $f : (X, w_2) \to (Y, v_1, v_2)$ *defined as* $f(a) = b$*,* $f(b) = a$*,* $f(c) = c$*. Then* f *is faintly* $(w_2, \Theta_{v_1 v_2})$*-continuous but is neither* $(\Theta_{w_1 w_2}, \Theta_{v_1 v_2})$*-continuous nor* $(\Theta_{w_2 w_1}, \Theta_{v_1 v_2})$*-continuous*

Example 9 *The function defined in Example 4 is not faintly* $(w_1, \Theta_{w_1 w_2})$*-continuous*

Remark 6 *If* (X, w_1, w_2)*,* (Y, v_1, v_2) *are two biweak spaces and* $f : (X, w_1, w_2) \to (Y, v_1, v_2)$ *is a function. The concepts of* $(\Theta_{w_1 w_2}, \Theta_{v_1 v_2})$*-continuous and faintly* $(w_1, \Theta_{v_1 v_2})$*-continuous are independent.*

Theorem 10 *Let* (X, w_1, w_2) *and* (Y, v_1, v_2) *be two biweak spaces. If* $f : (X, w_1, w_2) \to (Y, v_1, v_2)$ *is* $(\Theta_{w_1 w_2}, \Theta_{v_1 v_2})$*-continuous, then for every* $\Theta_{v_1 v_2}$*-closed set* F*,* $f^{-1}(F)$ *is a* $\Theta_{w_1 w_2}$*-closed set.*

Proof. It follows by duality. □

Definition 9 *Let* (X, w_1) *be a weak space and* (Y, v_1, v_2) *be a biweak space. A function* $f : (X, w_1) \to (Y, v_1, v_2)$ *is said to be mixed weakly* $(w_1, v_1 v_2)$*-continuous at* $x \in X$ *if for every* v_1*-open set* V*, containing* $f(x)$*, there exists a* w_1*-open set* U *containing* x *such that* $f(U) \subseteq v_2 C(V)$*. Then* f *is mixed weakly* $(w_1, v_1 v_2)$*-continuous if it is mixed weakly* $(w_1, v_1 v_2)$*-continuous at every point* $x \in X$*.*

Example 10 *Let* (X, w_1) *be a weak space and* (Y, ν_1, ν_2) *be a biweak space, where* $X = Y = \{a, b, c\}$ *and weak structures:* $w_1 = \{\emptyset, X, \{a\}, \{b\}\}$*,* $\nu_1 = \{\emptyset, X, \{b\}\}$ *and* $\nu_2 = \{\emptyset, X, \{a\}\}$*. Consider* $f : (X, w_1) \to (Y, \nu_1, \nu_2)$*, defined as* $f(a) = b$*,* $f(b) = c$*,* $f(c) = a$*. Then* f *is mixed weakly* $(w_1, \nu_1\nu_2)$*-continuous.*

Remark 7 *Let* (X, w) *be a weak space and* (Y, ν_1, ν_2) *be a biweak space. If* $\nu_1 = \nu_2$*, then the notion of mixed weakly* $(w, \nu_1\nu_2)$*-continuous function is just the notion of weak weakly* (w, ν_1)*-continuous functions, that is, for any* ν_1*-open set* V*, there exists a* w_1*-open set* U *such that* $f(U) \subseteq \nu_1 C(V)$*.*

Theorem 11 *Let* $f : X \to Y$ *be a function,* w_1 *a weak structure on a nonempty set* X*, and* ν_1, ν_2 *be two weak structures on a nonempty set* Y*. Then:*

1. *If* f *is mixed weakly* $(w_1, \nu_1\nu_2)$*-continuous, then* $f(w_1 C(A)) \subseteq \gamma\theta_{\nu_1\nu_2}(f(A))$ *for every subset* A *of* X*.*

2. *If* $f(w_1 C(A)) \subseteq \gamma\theta_{\nu_1\nu_2}(f(A))$ *for every subset* A *of* X*, then* $w_1 C(f^{-1}(\nu_2 I(G)) \subseteq f^{-1}(\nu_1 C(V))$ *for every* ν_2*-open set* V *of* Y*.*

Proof. 1. Consider $A \subseteq X$, $x \in w_1 C(A)$ and V any ν_1-open set containing $f(x)$. By hypothesis f is mixed weakly $(w_1, \nu_1\nu_2)$-continuous, then there exists a w_1-open set U containing x such that $f(U) \subseteq \nu_2 C(V)$. Since $x \in w_1 C(A)$ and U is a w_1-open set U containing x, $A \cap U \neq \emptyset$. In consequence, $\emptyset \neq f(A) \cap f(U) \subseteq \nu_2 C(V) \cap f(A)$. Follows that $f(x) \in \gamma\theta_{\nu_1\nu_2}(f(A))$ and hence, $f(w_1 C(A)) \subseteq \gamma\theta_{\nu_1\nu_2}(f(A))$.
2. Clear. □

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Received: May 7, 2018