Newtonian Heating Effects of Oldroyd-B Liquid Flow with Cross-Diffusion and Second Order Slip

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Abstract. The current study highlights the Newtonian heating and second-order slip velocity with cross-diffusion effects on Oldroyd-B liquid flow. The modified Fourier heat flux is included in the energy equation system. The present problem is modeled with the physical governing system. The complexity of the governing system was reduced to a nonlinear ordinary system with the help of suitable transformations. A homotopy algorithm was used to validate the nonlinear system. This algorithm was solved via MATHEMATICA software. Their substantial aspects are further studied and reported in detail. We noticed that the influence of slip velocity order two is lower than the slip velocity order one.

Keywords: Oldroyd-B liquid · Second order slip · Cross diffusion effects · Convective heating · Cattaneo-Christov heat flux

1 Introduction

Heat transport through non-Newtonian fluids is the significant study in recent times because of its industrial and engineering applications. Oldroyd-B fluid is one of the types of non-Newtonian fluids. This fluid contains viscoelastic behaviour. Loganathan et al. [1] exposed the 2nd-order slip phenomena of Oldroyd-B fluid flow with cross diffusion impacts. Hayat et al. [2] performed the modified heat flux impacts with multiple chemical reactions on Oldroyd-B liquid flow. Eswaramoorthi et al. [3] studied the influence of cross-diffusion on viscoelastic liquid induced by an unsteady stretchy sheet. Elanchezhian et al. [4] examined the important facts of swimming motile microorganisms with stratification effects on Oldroyd-B fluid flow. Loganathan and Rajan [5] explored the entropy effects of Williamson nanoliquid caused by a stretchy plate with partial

Y. Tan et al. (Eds.): ICSI 2020, LNCS 12145, pp. 661–668, 2020.
https://doi.org/10.1007/978-3-030-53956-6_61

slip and convective surface conditions. The innovative research articles on non-Newtonian fluid flow with different geometry's and situations are studied in ref's [6–10].

As far as our survey report the Newtonian heating effects along with slip order two on Oldroyd-B liquid flow is not examined yet. The present study incorporates the cross diffusion and modified Fourier heat flux into the problem. The eminent homotopy technique [11–13] is employed for computing the ODE system and the results are reported via graphs.

2 Modeling

We have constructed the Oldroyd-B liquid flow subjected to below stated aspects:

1. Incompressible flow
2. Second-order velocity slip
3. Magnetic field
4. Binary chemical reaction
5. Stretching plate with linear velocity.
6. Cross-diffusion effects
7. Modified Fourier heat flux

Figure 1 represents graphical illustration of physical problem. The governing equations are stated below:

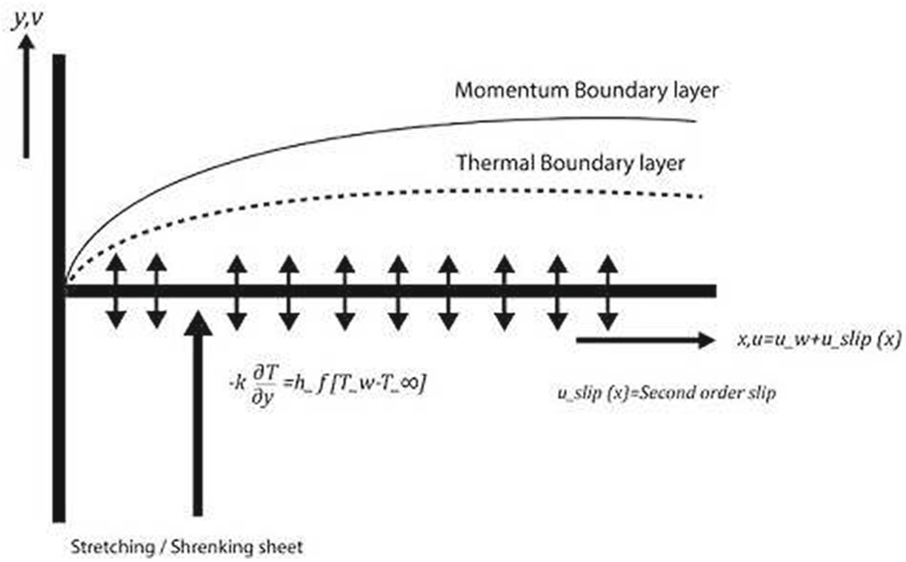

Fig. 1. Schematic diagram

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial x}u + \frac{\partial u}{\partial y}v + A_1\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2}$$
$$-\mu A_2 v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + \left(u\frac{\partial^3 u}{\partial x \partial y^2}\right) - \frac{\sigma B_0^2}{\rho}\left(u + A_1 v\frac{\partial u}{\partial y}\right), \tag{2}$$

$$\frac{\partial T}{\partial x}u + \frac{\partial T}{\partial y}v = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$\frac{\partial C}{\partial x}u + \frac{\partial C}{\partial y}v = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2} - k_m(C - C_\infty) \tag{4}$$

The boundary points are

$$u = u_w + u_{slip} = ax + \lambda_1\frac{\partial u}{\partial y} + \lambda_2\frac{\partial^2 u}{\partial y^2}, \quad v = 0,$$
$$-k\frac{\partial T}{\partial y} = h_f T, \quad C = C_w \quad \text{at } y = 0, \tag{5}$$
$$u(\to 0), \quad v(\to 0), \quad T(\to T_\infty), \quad C(\to C_\infty) \;\text{ as } y(\to \infty), \tag{6}$$

where A_1 (=relaxation time), A_2 (=retardation time), B_0 (=constant magnetic field), a (=stretching rate), c_p (=specific heat), c_∞ (=ambient concentration), c_w (=fluid wall concentration), D_m (=diffusion coefficient), k (=thermal conductivity), T_∞ (=ambient temperature), T_w (=convective surface temperature), u, v (=Velocity components), u_w (=velocity of the sheet), λ_1 (=first order slip velocity factor), λ_2 (=second order slip velocity factor), μ (=kinematic viscosity), ρ (=density), σ (=electrical conductivity), γ (=dimensionless thermal relaxation time). The energy equation updated with Cattaneo-Christov heat flux is defined as:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + 2uv\frac{\partial T^2}{\partial x \partial y} + \lambda\left(u^2\frac{\partial^2 T}{\partial x^2} + v^2\frac{\partial^2 T}{\partial y^2} + \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)\frac{\partial T}{\partial x}\right.$$
$$\left. + \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right)\frac{\partial T}{\partial y}\right) = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2} \tag{7}$$

The transformations are

$$\psi = \sqrt{a\mu}xf(\eta), u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}, \eta = \sqrt{\frac{a}{\mu}}y$$
$$v = -\sqrt{a\mu}f(\eta), \quad u = axf'(\eta), \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \tag{8}$$

From the above transformations we derive the ODE system as follows,

$$f''' + \beta\left(f''^2 - ff^{iv}\right) + ff'' - f'^2 + \alpha\left(2ff'f'' - f^2f'''\right) - M\left(f' - \alpha ff''\right) = 0 \quad (9)$$

$$f\theta' - \gamma\left(f^2\theta'' + ff'\theta'\right) + \frac{1}{Pr}(1 + \frac{4}{3}Rd)\theta'' + D_f\phi'' = 0 \quad (10)$$

$$\frac{1}{Sc}\phi'' + f\phi' - Cr\phi + Sr\theta'' = 0 \quad (11)$$

with boundary points

$$f(0) = 0,\ f'(0) = 1 + \epsilon_1 f''(0) + \epsilon_2 f'''(0),\quad \theta'(0) = -Nw(1 + \theta(0)),\quad \phi(0) = 1$$
$$f'(\infty) = 0,\quad \theta(\infty) = 0,\quad \phi(\infty) = 0, \quad (12)$$

The variables are defined as:
$\epsilon_1 =$ (first order velocity constant) $= \lambda_1\sqrt{a/\mu}$; $\epsilon_2 =$ (second order velocity constant) $= \lambda_2 \frac{a}{\mu}\frac{h_f}{k}\sqrt{\mu/a}$; $\alpha =$ (relaxation time constant) $= A_1 a$; $\beta =$ (retardation time constant) $= A_2 a$; M = (magnetic field constant) $= \frac{\sigma B_0^2}{\rho a}$; $Pr =$ (Prandtl number) $= \frac{\rho C_p}{k}$; $Rd =$ (radiation constant) $= \frac{4\sigma^* T_\infty^3}{kk^*}$; $\gamma = \lambda a$; $D_f =$ (Dufour number) $= \frac{D_m k_T}{\mu c_s c_p}\frac{c_w - c_\infty}{T_w - T_\infty}$; $Cr =$ (chemical reaction constant) $= \frac{k_m}{a}$; $Sc =$ (Schmidt number) $= \frac{\mu}{D_m}$; $Sr =$ (Soret number) $= \frac{D_m k_T}{\mu T_m}\frac{T_w - T_\infty}{c_w - c_\infty}$.

3 Solution Methodology

We using the homotopy technique for validate the convergence of the nonlinear systems. The basic guesses and linear operators are defined as:

$$f_0 = \eta e^{-\eta} + \frac{3\epsilon_2 - 2\epsilon_1}{\epsilon_2 - 1 - \epsilon_1} * e^{-\eta} - \frac{3\epsilon_2 - 2\epsilon_1}{\epsilon_2 - 1 - \epsilon_1}, \qquad \phi_0 = e^{-\eta}, \qquad \theta_0 = \frac{Nw * e^{-\eta}}{1 - Nw}$$
$$L_f = f'(f'' - 1), \qquad L_\phi = (\phi'') - (\phi), L_\theta = (\theta'') - (\theta).$$

which satisfies the property

$$L_f\left[D_1 + D_2e^\eta + D_3e^{-\eta}\right] = 0, \quad L_\phi\left[D_6e^\eta + D_7e^{-\eta}\right] = 0, \quad L_\theta\left[D_4e^\eta + D_5e^{-\eta}\right] = 0,$$

where $D_k(k = 1 - -7)$ are constants. The special solutions are

$$f_m(\eta) = f_m^*(\eta) + D_1 + D_2e^\eta + D_3e^{-\eta}$$
$$\phi_m(\eta) = \phi_m^*(\eta) + D_6e^\eta + D_7e^{-\eta}$$
$$\theta_m(\eta) = \theta_m^*(\eta) + D_4e^\eta + D_5e^{-\eta}.$$

In Fig. 2 the straight lines are named as h-curves. The permissible range of h_f, h_θ & h_ϕ are $-1.7 \leq h_f \leq -0.6, -1.2 \leq h_\theta \leq -0.2, -1.2 \leq h_\phi \leq -0.2$, respectively. Order of convergent series is depicted in Table 1. Table 2 depicts $f''(0)$ in the special case $M = \beta = 0$. It is noted that the $f''(0)$ values are well matched with the previous reports [14–16].

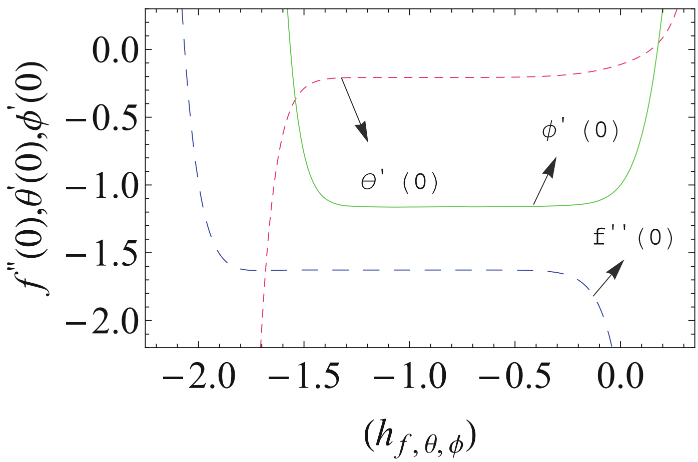

Fig. 2. h-curves for h_f, h_θ, h_ϕ

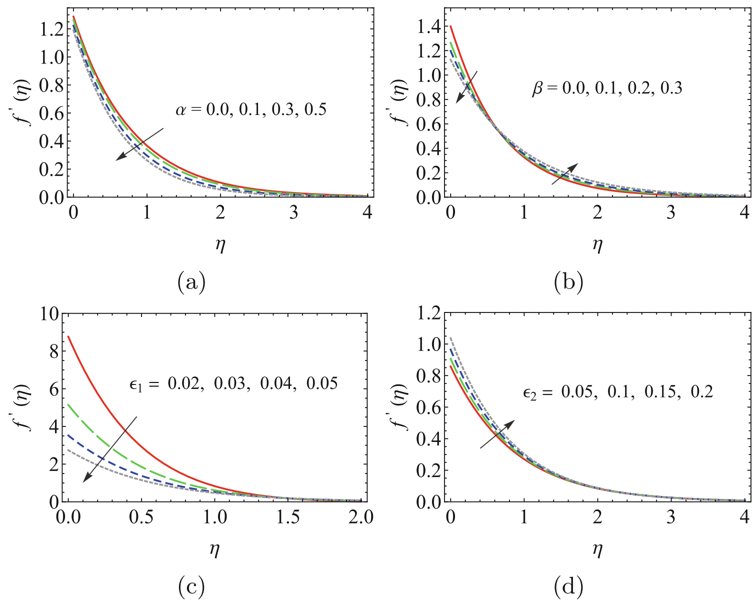

Fig. 3. $f'(\eta)$ for various range of parameters (α, β, ϵ_1, ϵ_2).

4 Results and Discussion

Physical Characteristics of rising parameters versus, Concentration $\phi(\eta)$, velocity $f(\eta)$ and temperature $\theta(\eta)$ are investigated in Figs. 3, 4 and 5. Figure 3 depicted the velocity distribution $f(\eta)$ for different range of α, β, ϵ_1, ϵ_2. It is noted that the velocity reduces for β and ϵ_1, while it increases for α and ϵ_2. The temperature

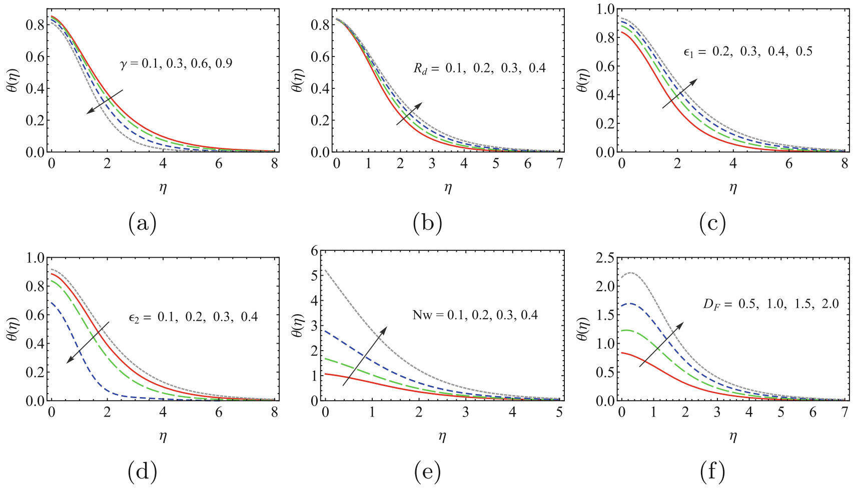

Fig. 4. $\theta(\eta)$ for various range of parameters (γ, R_d, ϵ_1, ϵ_2, Nw and D_F).

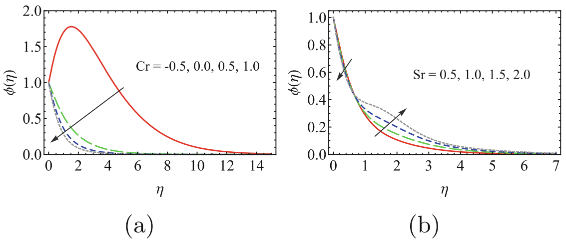

Fig. 5. $\phi(\eta)$ for various range of parameters (Cr and Sr).

Table 1. Approximations for convergence

Order	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
2	1.8126	0.1532	1.1289
7	1.6331	0.1979	1.1558
12	1.6273	0.2059	1.1606
17	1.6274	0.2068	1.1616
22	1.6274	0.2068	1.1616
27	1.6274	0.2068	1.1616
35	1.6274	0.2068	1.1616

distribution $\theta(\eta)$ for different range of γ, R_d, ϵ_1, ϵ_2, Nw and D_F are sketched in Fig. 4. Thermal boundary layer decays with increasing the γ and ϵ_2 values. Larger values of R_d, ϵ_1 and D_F boosts the temperature distribution $\theta(\eta)$. Figure 5

Table 2. Validation of $f''(0)$ in the specific case for various α when $\beta = M = 0$

α	Ref. [14]	Ref. [15]	Ref. [16]	Present
0.0	1.000	0.9999963	1.00000	1.00000
0.2	1.0549	1.051949	1.05189	1.05189
0.4	1.10084	1.101851	1.10190	1.10190
0.6	1.0015016	1.150162	1.15014	1.15014
0.8	1.19872	1.196693	1.19671	1.19671

shows the influence on $\phi(\eta)$ for various values of Cr and Sr. These parameters shows the opposite effect in $\phi(\eta)$.

5 Conclusion

The salient outcomes the flow problem is given below:

1. Retardation time parameter (β) is inversely proportional to the relaxation time parameter (α) is in velocity profile.
2. Thermal boundary layer enhances due to increasing the R_d,Nw, D_F whereas it decays for higher ϵ_1 and γ.
3. Higher Soret number values enhance the solutal boundary thickness.

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