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ELEMENTS OF A GRAVITATIONAL LENS SYSTEM ASSUMING AN ELLIPTICAL GALAXY MODEL

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ABSTRACT

This work studies some elements of gravitational lensing by galaxies such as the lens equation, deflection angle, lensing potential and time delay, modeling the mass distribution of the lensing galaxy as an elliptical galaxy. The mass distribution function ρ of the deflecting galaxy indicates that it has a nucleus with radius a in its core, a free-form parameter b ($b > a$), and that the mass density of the nucleus is ρ_0. The mass density distribution ρ allows us to find the surface mass density Σ (projected on the plane of the lens), followed by general elements of the gravitational lens expressed in terms of the geometric parameters a and b. The relation between these parameters is defined by the adimensional factor $n = b/a > 1$. The results of this work can be applied to any galactic lens system to conduct an analysis based on the temporal delay between two images and to determine the conditions that must be satisfied by the parameter n.

RESUMEN

En este trabajo se estudian algunos elementos de las lentes gravitacionales causadas por galaxias como son ecuación de la lente, ángulo de desviación, potencial de desviación y retardo temporal, modelando la distribución volumétrica de masa de la lente en forma elíptica. La función de distribución volumétrica de masa en la galaxia deflectora ρ, contiene en su centro un núcleo de radio a, una densidad volumétrica en su núcleo ρ_0 y un parémetro libre de forma b ($b > a$). Mediante la distribución de densidad volumétrica de masa se encuentra inicialmente la densidad superficial de masa Σ, (proyectada en el plano de la lente), para después hallar los elementos de una lente gravitacional, los cuales son totalmente generales y quedan en términos de los parámetros geométricos a and b, que se han relacionado mediante un factor adimensional $n = b/a > 1$. Los resultados encontrados se aplican a cualquier sistema de lentes causado por una galaxia, para hacer un análisis basado en el retardo temporal entre dos imágenes y ver las condiciones que debe cumplir el parámetro n.

Key Words: galaxies: elliptical and lenticular — gravitation — gravitational lensing: strong — gravitational lensing: weak

1. INTRODUCTION

In the study of gravitational lenses (GL), the distribution of mass density of the deflector (ρ) can be projected onto a plane perpendicular to the line of sight between the observer and the light source. This plane is called the lens plane, according to Narayan & Bartelmann (1997). The lens is assumed to be thin and the distribution of the lens mass is substituted by a plane on which the surface mass density is (Σ), resulting in the so-called thin lens approximation, according to Schneider et al. (1992).

In general, GL are characterized by a few basic elements such as: surface mass density (Σ), lens equation, deflection angle (α), lensing potential (Ψ) and time delay (Δt), which constitute a set of basic tools that can be used for the study of various types of lens systems. The analytical expressions that describe these elements can be used to study specific mass distribution models of a particular galaxy.

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Some observational data show that astronomical objects that act as lenses can be modeled in different ways, according to Cohen & Hewitt (2000). In gravitational lens systems produced by galaxies, some parameters can be observationally measured, such as: dispersion velocity of the material particles that constitute the lens (σ_p), angular position of the images (θ), red shifts of the lens (z_L), light source (z_S) and time delay (Δt). These parameters allow the study and modelling of gravitational lens systems.

In these systems the distances from the light source to the lens and from the lens to the observer are $\approx 1\,pc$. Thus, the elements that constitute a lens system, such as the deflecting galaxy, the light source and the observer, are too far away, so that light travels in free space most of the time, and is deviated only when it passes through the lens. To model a lens system, it is thus necessary to model the universe through which the ray of light passes; to do this we need to choose cosmological parameters, such as those used by Adler et al. (1975), Ciufolini et al. (1995), Foster et al. (1994) and Kenion (1995): vacuum density (Ω_v); matter density (Ω_m); softness parameter ($\widetilde{\alpha}$) and Hubble constant (H_0).

Using observational values and specifying these cosmological parameters makes it possible to apply the general properties of GL to specific lens systems, with different galactic models.

In this work, we assume a model of a galaxy with an elliptical mass distribution (ρ), which we use to determine the analytical expressions that describe the elements of GL according to Brainerd et al. (1996) and Golse et al. (2002). Here we follow Merritt & Valluri (1997) and Molina et al. (2006), who propose a distribution of mass density useful for models of elliptical galaxies acting as gravitational lenses.

2. LENS COMPONENTS AND ELLIPTICAL MODEL OF THE DEFLECTING GALAXY

2.1. *Lens Elements*

In the literature about GL, the approximation of a flat lens is characterized by a surface mass density given by the projection operator, according to Miranda, Molina, & Viloria (2014),

$$\Sigma(\vec{R}) = \int \rho(\vec{R}, z)dz, \tag{1}$$

where $\vec{R}$ is a radius vector in the lens plane as shown in Figure 1, and ρ is the mass distribution of the lens. The radius vector R, called impact parameter, can be written as $R = \xi_0 x$. The quantity ξ_0 is known as

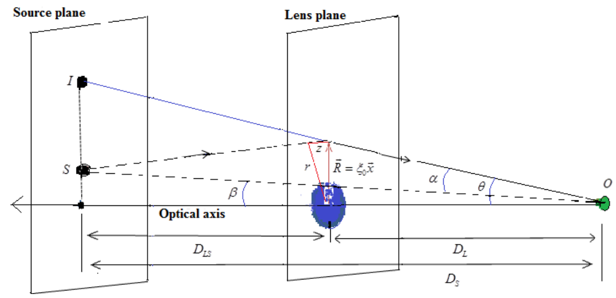

Fig. 1. Illustration of a gravitational lens system. The angular separations of the source and the image from the optical axis as seen by the observer are β and θ, respectively. The angular diameter distances between the observer and the source, the observer and the lens, and the lens and the source are D_S, D_L and D_{LS}, respectively. See Narayan et al. (1997).

a scale parameter or scale factor, and is defined according to the lens model being used. In this work, it is defined below in equation (10). According to Schneider et al. (1992, p. 231), the matter within the disc of radius x around the center of mass contributes to the deflection of the ray of light, while the matter outside the disc ($x' < x$) does not contribute importantly to the deflection; thus, the deflection angle can be expressed as:

$$\alpha(x) = \frac{2}{x}\int x'\kappa(x')dx', \tag{2}$$

where $x' < x$. The quantity κ defined as $\kappa(x) = \Sigma(x)/\Sigma_{cr}$ is the so-called convergence, which indicates the existence of a minimal or critical surface density for the GL phenomenon to occur, according to Narayan et al. (1997) and Schneider et al. (1992). We define the critical mass surface density as: $\Sigma_{cr} = c^2 D_S/4\pi \times G \times D_L \times D_{LS}$, where D_S, D_L and D_{LS}, are the angular diameter distances between observer-source, observer-lens and lens-source, respectively, as shown in Figure 1.

The lensing potential, according to Narayan et al. (1997, p. 19) is defined by:

$$\psi = \frac{1}{\pi}\int \kappa(x')\ln|x - x'|\,dx'^2\,. \tag{3}$$

The time delay Δt between two light beams detected by an observer is given by Narayan et al. (1997, p. 20) as:

$$\Delta t = \frac{(1+z_L)}{c}\frac{\xi_0^2 D_S}{D_L D_{LS}}\left(\frac{1}{2}\left[\alpha_2^2 - \alpha_1^2\right] - \left[\psi_2 - \psi_1\right]\right), \tag{4}$$

where z_L is the redshift of the lensing galaxy, D_L is the angular diameter distance between observer and lens, D_S is the angular diameter distance between observer and source, and D_{LS} is the angular diameter distance between lens and source.

Expression (4) contains the geometric delay described by equation (2), and the gravitational potential given by equation (3).

Furthermore, the relationship between the source position (β), the positions of the images ($\theta = \xi/D_L$) and the deflection angle can be writen according to equation (2.15a) in Schneider et al., (1992, p.31) as:

$$\beta = \frac{\xi_0}{D_L}x - \frac{D_{LS}}{D_S}\alpha(x), \tag{5}$$

which is called the lens equation. Equations (2), (3), (4) and (5) constitute the set of basic elements required for the study of GL.

2.2. *Model of an Elliptical Lensing Galaxy*

For this study, we modeled the distribution of the lens mass as an elliptical galaxy, following Merritt & Valluri (1996) and Molina et al. (2006), who proposed a distribution of mass density that fits elliptical galaxy models acting as gravitational lenses. The analytical expressions of GL were developed based on this mass distribution. This distribution of mass can be written as:

$$\rho(r) = \frac{\rho_0}{\left(1+\frac{r^2}{a^2}\right)\left(1+\frac{r^2}{b^2}\right)}\,. \tag{6}$$

The model of a lensing galaxy contains a central nucleus with radius a, a free-form parameter acting as a scale factor b ($b > a$), and a density of the mass in the nucleus ρ_0. Introducing the mass density given in equation (6) into the projection operator defined in equation (1), and substituting the variables $z^2 = r^2 - R^2$ (see Figure 1), we obtain the surface mass density of the lens. The distance z is typically much smaller than the distances between observer and lens and between lens and source; thus after evaluating the integrals, the surface mass density of the lens takes the following form,

$$\Sigma(n,R) = \frac{\sum_0 n^2 a}{n^2-1}\left[\frac{1}{\sqrt{a^2+R^2}} - \frac{1}{\sqrt{n^2a^2+R^2}}\right], \tag{7}$$

where $n = b/a > 1$ is the adimensional parameter and $\Sigma_0 = \pi\rho_0 a$ is the surface density of the nucleus.

To find the analytical expressions of the deflection angle and the lensing potential, we first substitute $R = \xi_0 x$, in equation (7), so that the surface density of the flat lens is:

$$\Sigma(n,x) = \frac{\sum_0 n^2 a}{(n^2-1)\xi_0}\left[\frac{1}{\sqrt{A^2+x^2}} - \frac{1}{\sqrt{n^2A^2+x^2}}\right], \tag{8}$$

where $A = a/\xi_0$ a new parameter. The convergence factor $\kappa = \Sigma/\Sigma_{cr}$ (defined above), now has the form:

$$\kappa(n,x) = \frac{1}{2}\left[\frac{1}{\sqrt{A^2+x^2}} - \frac{1}{\sqrt{n^2A^2+x^2}}\right], \tag{9}$$

and the chosen scale factor is:

$$\xi_0 = \frac{8\pi G D_L D_{LS}\Sigma_0 n^2 a}{c^2 D_S(n^2-1)}\,. \tag{10}$$

The scale factor is based on the velocity dispersion of the components of the lensing galaxy σ_p (this is explained in the following paragraphs). The scale factor is expressed in terms of the central radius a, and the adimensional parameter $n = b/a > 1$. When the adimensional parameter approaches 1, that is, $n \approx 1$, the scale factor becomes infinite, which suggest that it would be preferable to select a scale that depended only on the radius a of the nucleus.

The time delay expressed in equation (4) contains the arbitrary scale factor ξ_0, that we chose for the elliptical distribution of mass, as shown in equation (10). The central mass density ρ_0 cannot be observationally measured in a gravitational lens system, but it can be determined from the velocity dispersion σ_p of the matter particles in the lensing galaxy. This dispersion can be determined using the expression proposed by Molina et al. (2006) and Tremaine et al. (1994):

$$\sigma_p^2 = \frac{2G}{\Sigma(R)}\int_R^\infty \frac{M(r)}{r^2}\rho(r)\sqrt{r^2-R^2}dr\,. \tag{11}$$

Expression (11) shows that the central mass density can be established if the velocity dispersion of the matter particles in the lensing galaxy is known; this, in turn, is used to determine the scale factor (10).

2.3. *Deflection Angle*

Expressed in terms of the impact parameter $R = \xi_0 x$ and using the convergence factor in equation (9), the deflection angle (2) takes the form,

$$\alpha(n,R) = \frac{(n-1)a}{R} + \sqrt{1+\frac{a^2}{R^2}} - \sqrt{1+\frac{n^2a^2}{R^2}}, \tag{12}$$

which is written in terms of the central radius a and the adimensional parameter $n = b/a > 1$. It can be seen that this parameter is close to 1, $n \approx 1$, so that, after selecting a scale factor, the deflection angle becomes $\alpha(R) \approx 0$.

2.4. *Lensing Potential*

The lensing potential can be found by substituting (9) into equation (3) and writing the expression in terms of the impact parameter $R = \xi_0 x$,

$$\begin{aligned}\psi(\mathbf{n}, R) = &\frac{a}{\xi_0}\left[(n-1) + \ln\frac{2a}{\xi_0} - n\ln\frac{2na}{\xi_0} + \sqrt{1+\frac{R^2}{a^2}}\right.\\ &-\sqrt{n^2+\frac{R^2}{a^2}} - \ln\frac{a(1+\sqrt{1+\frac{R^2}{a^2}})}{\xi_0}\\ &\left.+n\ln\frac{a(n+\sqrt{n^2+\frac{R^2}{a^2}})}{\xi_0}\right],\end{aligned} \tag{13}$$

given that $n > 1$. If this adimensional parameter is close to 1, $n \approx 1$, and if a scale factor is selected, the lensing potential becomes zero, $\psi(R) \approx 0$.

2.5. *Time Delay*

Differentiating the square of the deflection angle of two images, $\alpha_2^2 - \alpha_1^2$, then differentiating the lensing potential $\psi_2 - \psi_1$, and substituting these derivatives into equation (4), we obtain the time delay as:

$$\Delta t = \frac{(1+z_L)\xi_0^2 D_S}{c D_L D_{LS}}\left[\frac{h(n,a)}{2} - a\frac{g(n,a)}{\xi_0}\right]. \tag{14}$$

Equation (13) allows us to determine the time delay between two images, where a is the radius of the nucleus of the lensing galaxy and n is an adimensional parameter.

Furthermore, we have defined below two new functions: $h(n,a)$ and $g(n,a)$, which depend on how the nucleus radius a is set and on how much these two new functions vary from the adimensional parameter n.

Function $h(n,a)$ is written as:

$$\begin{aligned}h(n,a) \;=\; &(n^2-n+1)\left(\frac{a^2}{R_2^2} - \frac{a^2}{R_1^2}\right)\\ &+(n-1)\left[\sqrt{\frac{a^2}{R_2^2}+\frac{a^4}{R_2^4}} - \sqrt{\frac{a^2}{R_2^2}+\frac{n^2a^4}{R_2^4}}\right]\\ &+(n-1)\left[\sqrt{\frac{a^2}{R_1^2}+\frac{n^2a^4}{R_1^4}} - \sqrt{\frac{a^2}{R_1^2}+\frac{a^4}{R_1^4}}\right]\\ &-\sqrt{1+\frac{a^2}{R_2^2}+n^2\left(\frac{a^2}{R_2^2}+\frac{a^4}{R_2^4}\right)}\\ &+\sqrt{1+\frac{a^2}{R_1^2}+n^2\left(\frac{a^2}{R_1^2}+\frac{a^4}{R_1^4}\right)}.\end{aligned} \tag{15}$$

Similarly, $g(n,a)$ is defined as:

$$\begin{aligned}g(n,a) \;=\; &\sqrt{1+\frac{R_2^2}{a^2}} - \sqrt{1+\frac{R_1^2}{a^2}}\\ &+\sqrt{n^2+\frac{R_1^2}{a^2}} - \sqrt{n^2+\frac{R_2^2}{a^2}}\\ &+\ln\left|\frac{1+\sqrt{1+\frac{R_1^2}{a^2}}}{1+\sqrt{1+\frac{R_2^2}{a^2}}}\right|\\ &+n\ln\left|\frac{n+\sqrt{n^2+\frac{R_2^2}{a^2}}}{1+\sqrt{n^2+\frac{R_1^2}{a^2}}}\right|.\end{aligned} \tag{16}$$

$h(n,a)$ and $g(n,a)$ are normalized so that when they are introduced into the time delay equation (14), their units are expressed in seconds.

2.6. *Lens Equation*

By substituting the deflection angle (12) into equation (5), we get,

$$\begin{aligned}\beta \;=\; &\frac{R}{D_L} - \frac{D_{LS}}{D_S}\frac{(n-1)a}{R}\\ &-\frac{D_{LS}}{D_S}\left[\sqrt{1+\frac{a^2}{R^2}} - \sqrt{1+\frac{n^2a^2}{R^2}}\right].\end{aligned} \tag{17}$$

This expression represents the lens equation in the elliptical lens model we propose; it is a function of the impact parameter and the adimensional parameter n.

3. APPLICATION OF THE PROPOSED GRAVITATIONAL LENS MODEL TO THE GRAVITATIONAL LENS B0218 + 357

The expressions obtained in the previous section are completely general and can be applied to any gravitational lens system. By way of example, we choose to apply them to the B0218 + 357 lens system in order to asses the consistency of the results obtained.

Some researchers, including, Wucknitz et al. (2004), have determined the Hubble constant H_0 from the study of B0218 + 357 and discussed different models of this system. To obtain an estimate of the Hubble constant Biggs et al. (1999) modeled the B0218 + 357 system using the lens model used by Kormann et al. (1994), in which the lens system is described as a singular isothermal ellipsoid (SIE). More information on the morphology of the

TABLE 1

DATA OBSERVED FOR THE B0218 +357 LENS SYSTEM

σ_p km/s	θ_1 mas	θ_2 mas	$z_{\rm L}$	$z_{\rm S}$	Δt days
150	40	290	0.68	0.94	10.5 ± 0.2

observed images can be found in the work of Spingola et al. (2015). Observational data for the lens system B0218 + 357 obtained from the CASTLES Survey by Cohen & Hewitt (2000) is summarized in Table 1: the velocity dispersion σ_p, the angular positions of two images θ_1 and θ_2, the redshifts for the lens z_L and the source z_S, and the difference in time delay between the two images Δt.

The cosmological parameters chosen for this work are within the range of values most widely accepted in the literature. We relied on the work of several authors, and especially Kessler et al. (2009), Boughn & Crittenden (2001), Bartelmann et al. (1997), Weinberg (1972), Grogin et al. (1996) and others. The parameters chosen for our elliptical lens model are as follows: Hubble constant, $H_0 = 76\,(\text{km/s})\text{pc}^{-1}$, vacuum density $\Omega_v = 0.7$, matter density $\Omega_d = 0.3$, and softness parameter $\widetilde{\alpha} = 0.5$. This softness parameter of the matter in the universe is smoothly distributed (i.e., it is not bound up in galaxies), according to Dyer & Roeder (1973), see also P. Schneider et al. (1992, p.138) and Xi Yang et al. (2013).

According to Dyer (1973) and Wucknitz et al. (2004), using these cosmological parameters for the gravitational lens system B0218 + 357 and the values for the cross section $D_L D_{LS}/D_S$, the observer-lens distance D_L, and the impact parameters of the two images R_1 and R_2, we obtain: $D_L D_{LS}/D_S = 2.25 \times 10^7$ pc, $D_L = 1.364 \times 10^9$ pc, $R_1 = 264.58$ pc and $R_2 = 1918.21$ pc.

From these values, it is possible to obtain the elements of the proposed lens model.

3.1. *Scale Factor, Surface Mass Density of the Lens, Deflection Angle and Lensing Potential*

Since there are two images in the lens system B0218 + 357, the previous calculations of the two impact parameters $R_1 = 264.58$ pc and $R_2 = 1918.21$ pc allow us to set the radius of the nucleus within the range of values of these two parameters (R_1 and R_2). In this work, we set the radius of the nucleus at approximately the value of the smaller impact parameter, $a = 264$ pc, which corresponds to the impact parameter with the largest deflection. To facilitate the analysis, we define the adimensional quantity λ as $\lambda = R/a > 1$. By setting the radius of the nucleus and varying the impact parameter according to it, we can also set the surface density of the nucleus at the approximate value of $\Sigma_0 = 58.62\text{kg/m}^2$.

Furthermore, the scale factor in equation (10) is given in terms of the adimensional parameter n, that is,

$$\xi_0 = (200.16\,\text{pc})\frac{n^2}{(n^2-1)}\,, \tag{18}$$

where $1\text{pc} = 3.086 \times 10^{16}$ m. The surface mass density of the lens, expressed in equation (8) is then found to be

$$\Sigma(n,\lambda) = \frac{58.62n^2}{n^2-1}\left[\frac{1}{\sqrt{1+\lambda^2}} - \frac{1}{\sqrt{n^2+\lambda^2}}\right]\text{kg/m}^2 \tag{19}$$

where, for this particular lens, the impact parameter satisfies the condition $1 \le \lambda \le 7.25$ and $n > 1$. When the value of n is fixed, expression (19) allows us to estimate the surface mass density of the lens as a function of λ, in the given interval.

After making the substitutions required by our proposed model, the deflection angle described by equation (12) takes the form:

$$\alpha(n,\lambda) = \frac{n-1}{\lambda} + \sqrt{1+\frac{1}{\lambda^2}} - \sqrt{1+\frac{n^2}{\lambda^2}}\,, \tag{20}$$

for which we know that $1 \le \lambda \le 7.25$ and $n > 1$.

At the same time, the lensing potential expressed in equation (13) takes the new form:

$$\begin{aligned}\psi(\mathbf{n},\lambda) \quad = \quad & \frac{a}{\xi_0}\Bigg[(n-1) + \ln\frac{2a}{\xi_0} - n\ln\frac{2na}{\xi_0} \\ & +\sqrt{1+\lambda^2} - \sqrt{n^2+\lambda^2} \\ & -\ln\frac{a(1+\sqrt{1+\lambda^2})}{\xi_0} \\ & +n\ln\frac{a(n+\sqrt{n^2+\lambda^2})}{\xi_0}\Bigg]\,,\end{aligned} \tag{21}$$

which depends on the scale factor defined in equation (18) and on the following conditions: $1 \le \lambda \le 7.25$ and $n > 1$.

3.2. *Time Delay Model*

By using the values obtained from the B0218 + 357 lens system, the time delay stated in

TABLE 2

TIME DELAY VALUES IN THE LENS SYSTEM B0218 + 357

n	2	2.1	2.2
Δt (days)	9.67	10.3	10.9

equation (14) is reduced to

$$\Delta t = (5.68\,\mathrm{days})\frac{n^2}{(n^2-1)^2} \times \left[\frac{h(n)}{2} - \frac{1.32(n^2-1)g(n)}{n^2}\right] . \quad (22)$$

The values h and g, and equations (15) and (16), depend only on the adimensional parameter n. This allows us to establish values for the time delay. We took advantage of the fact that the time delay between the two images is observationally measured, as shown in Table 1 (10.5 days), and introduced different values for n in equation (22), until we reached the observed value. Table 2 shows the time delay given by equation (22) for some values of n.

When we compare the observational time delay $\Delta t = 10.5 \pm 0.2 days$ (Table 1) with equation (22), we see that the adimensional parameter has a range of certainty of $2 \leq n \leq 2.2$ for the lens system B0218 + 357.

Establishing the value of the parameter n allowed us to estimate the geometric parameters a and b of the elliptical lens and then to find the numerical values for the other basic elements of the lensing system, such as the surface density of the lens, the deflection angle, the scale factor and the lensing potential, as indicated in the following section.

3.3. *Estimation of the Deflection Angle, Lensing Potential and Lens Equation*

Given that for the B0218 + 357 lens system, the approximate radius of the nucleus is $a = 264\,\mathrm{pc}$, the adimensional parameter is $n = 2.1$, the cross section is $D_L D_{LS}/D_S$ and the observer-lens distance is D_L, we can estimate the deviation angle, deflection potential and the lens equation in this model:

1. Using as our premise equation (19), and considering that the value of the adimensional parameter is $n = 2.1$, and that the surface density decreases as the impact parameter increases, within the range $1 \leq \lambda \leq 7.25$, then the estimated surface mass density of the lens for the proposed system is in the range of $0.32\mathrm{kg/m}^2 \leq \Sigma \leq 32\mathrm{kg/m}^2$.

2. In accordance with equation (20), and considering that the parameter λ is in the range $1 \leq \lambda \leq 7.25$, we deduce that the deflection angle ranges between 120 mas and 220 mas.

3. In accordance with equation (18) and with the values of the adimensional parameter n shown in § 3.1, we can establish that the approximate value of the scale factor is $\xi_0 = 260\,\mathrm{pc}$. Furthermore, since the values of the parameter λ range between $1 \leq \lambda \leq 7.25$, the values of the lensing potential, according to equation (21), must range between 150 mas and 930 mas.

4. CONCLUSIONS

This work is based on a volumetric mass distribution that describes a fast relaxation scenario, similar to a model of an isothermal sphere, according to Kenyon (1990), where the mass of the lens is considered to be spherically symmetric. The model of an elliptical lensing galaxy includes a central core with radius a, a mass density in the central core ρ_0 and also a free shape parameter b ($b > a$). This mass distribution allowed us to find new analytical expressions of the lens elements. These new elements are equations for the lens surface density, the deviation angle, the deflection potential and the time delay. These expressions depend on the impact parameter of the images and on the geometric lens elements a and b, related by the adimensional parameter $n = b/a > 1$.

The analytical expressions that describe the surface mass density of the lens, the deflection angle, lensing potential and time delay of our proposed model can be used to analyze other galaxy lens systems. Our equations are quite general and applying them to study a specific lens system requires only the observational measurements indicated in Table 1.

The results of § 2 can be applied to any galaxy lens system whose mass density distribution fits the elliptical model we describe in this paper. The analytical expressions found in this work are a good point of departure for further research. In this work the proposed gravitational lens model is applied to the lens system B0218 + 357, using the observational values shown in Table 1. The values of the cosmological parameters used for our model are the most widely accepted in the literature, e.g. Bartelmann et al. (1997), Boughn et al. (2001), Foster et al. (1994), Kessler et al. (2009), Schneider et al. (1992), for the the Hubble constant, vacuum density, matter density and softness parameter. These cosmological parameters are reflected in the angular diameter distances between observer and lens, observer and - source and lens and source. Based on these values, we find the approximate radius of the nucleus of the lensing galaxy, the angular diameter distances, the scale factor defined by equation (18) and the adimensional parameter n.

The adimensional parameter n is adjusted to the time delay defined by equation (22), and compared to the observed time delay $\Delta t_{obs} = 10.5\,$days, shown in Table 1. To do this, it was necessary to write the impact parameter R, in terms of the radius of the nucleus $\lambda = R/a \geq 1$.

Because the impact parameter is expressed in terms of the radius of the nucleus, we were able, once the cosmological parameters were set, to determine the theoretical time delay given by equation (22), which depends only on the adimensional parameter n. Thus, when we compared the theoretical time delay given by equation (22) with the observationally measured time delay in Table 1, we found that the range of values of the adimensional parameter n is $2 \leq n \leq 2.2$.

Finally, as evidenced in § 3.3, using the value $n = 2.1$ for the adimensional parameter and the interval $1 \leq \lambda \leq 7.25$ for the impact parameter allowed us to estimate numerical values for the surface density of the lens, the deflection angle, the scale factor and the lensing potential for our proposed model.

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