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On $\mathcal{LC}$-sets and decomposition of continuity via bioperations

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Abstract

The aim of the present paper is to introduce and study the notions of $\mathcal{LC}$-set and $\mathcal{LC}$-continuity via biopeation. We discuss some properties of these notions.

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1. Introduction

Several generalizations or extensions derived from the classical notion of open set studied in General Topology are now the research topics of many topologists worldwide. Indeed a large number of interesting works in General Topology and Real analysis concerns the various modified forms of continuity, separation axioms etc. by using different classes of generalized open sets. Kasahara [1] introduced the concept of an operation on topological spaces. Umehara et al. [5] defined the notion of $\tau_{(\gamma,\gamma')}$ which is the collection of all (γ,γ')-open sets in a topological space (X,τ). More rencently, Carpintero et al. [3] introduced some new types of sets via bioperations and obtained a new decomposition of bioperation-continuity. In this paper, we introduce some new types of sets in bioperation-topological space. Also we discuss some properties and characterization of these new notions. The aim of the present paper is to introduce and study notions of $\mathcal{LC}$ -set and $\mathcal{LC}$ -continuity via bioperations.

2. Preliminaries

In this section we establish the basic notions to use in this paper. We refer to [2] for more details about notations and terminologies. However, we give the following preliminaries definitions.

Definition 2.1 : [1] Let (X,τ) be a topological space. An operation γ on the topology τ is function from τ into the power set $\mathcal{P}(X)$ of X such that $V \subset V^{\gamma}$ for each $V \in \tau$, where V^{γ} denotes the value of τ at V. It is denoted by $\gamma : \tau \to \mathcal{P}(X)$.

A bioperation-topological space is denoted by (X,τ,γ,γ'), where (X,τ) is a topological space equipped with two operations, say, γ and γ' defined on τ.

Definition 2.2 : A subset A of a bioperation-topological space (X,τ,γ,γ') is said to be (γ,γ')-open set [5] if for each $x \in A$ there exist open neighborhoods U and V of x such that $U^{\gamma} \cup V^{\gamma'} \subset A$. The complement of a (γ,γ')-open set is called a (γ,γ')-closed set. $\tau_{(\gamma,\gamma')}$ denotes set of all (γ,γ')-open sets in (X,τ,γ,γ').

Definition 2.3 : [5] For a subset A of a bioperation-topological space (X,τ,γ,γ'), we have the following

(1) $(\gamma,\gamma')\text{-}Cl(A) = \cap\{F : A \subset F, X \setminus F \in \tau_{(\gamma,\gamma')}\}$.

(2) $(\gamma,\gamma')\text{-}Int(A) = \cup\{U : U \subset A, U \in \tau_{(\gamma,\gamma')}\}$.

Definition 2.4 : Let (X,τ) be a topological space and A be a subset of X and γ and γ' be operations on τ. Then A is said to be

(1) $(\gamma,\gamma')\text{-}\alpha$-open if $A \subset (\gamma,\gamma')\text{-}Int((\gamma,\gamma')\text{-}Cl((\gamma,\gamma')\text{-}Int(A)))$,

(2) (γ,γ')-semi-open if $A \subset (\gamma,\gamma')\text{-}Int((\gamma,\gamma')\text{-}Cl((\gamma,\gamma')\text{-}Int(A)))$,

(3) (γ,γ')-preopen [2] if $A \subset (\gamma,\gamma')\text{-}Int((\gamma,\gamma')\text{-}Cl(A))$,

(4) (γ,γ')-semi-preopen if $A \subset (\gamma,\gamma')\text{-}Cl((\gamma,\gamma')\text{-}Int((\gamma,\gamma')\text{-}Cl(A)))$,

(5) (γ,γ')-regular open if $A = (\gamma,\gamma')\text{-}Int((\gamma,\gamma')\text{-}Cl(A))$,

(6) $(\gamma,\gamma')\text{-}\mathcal{A}$-set if $A = G \cap H$, where $G \in \tau_{(\gamma,\gamma')}$ and H is a (γ,γ')-regular closed set.

(7) (γ,γ')-t-set [4] if $(\gamma,\gamma')\text{-}Int((\gamma,\gamma')\text{-}Cl(A)) = \gamma,\gamma')\text{-}Int(A)$,

(8) $(\gamma,\gamma')\text{-}\mathcal{B}$-set [4] $A = G \cap H$, where $G \in \tau_{(\gamma,\gamma')}$ and H is a t-set.

The complement of (γ,γ')-regular open set is called a (γ,γ')-regular closed set. The family of all (γ,γ')-open (resp. $(\gamma,\gamma')\text{-}\alpha$-open, (γ,γ')-semi-open, (γ,γ')-preopen) sets of (X,τ,γ,γ') is denoted by $\tau_{(\gamma,\gamma')}$ (resp. $\alpha\tau_{(\gamma,\gamma')}$, $s\tau_{(\gamma,\gamma')}$, $p\tau_{(\gamma,\gamma')}$). The set of all $(\gamma,\gamma')\text{-}\mathcal{A}$-sets of (X,τ,γ,γ') is denoted by $\mathcal{A}$.

Definition 2.5 : A function $f : (X,\tau,\gamma,\gamma') \to (Y,\sigma,\beta,\beta')$ is said to be

(1) $(\gamma,\gamma')\text{-}(\beta,\beta')$-continuous if $f^{-1}(V)$ is (γ,γ')-open in X for every $V \in \sigma_{(\beta,\beta)'}$,

(2) $(\gamma,\gamma')\text{-}(\beta,\beta')\text{-}\alpha$-continuous if $f^{-1}(V)$ is $(\gamma,\gamma')\text{-}\alpha$-open in X for every $V \in \sigma_{(\beta,\beta)'}$,

(3) $(\gamma,\gamma')\text{-}(\beta,\beta')$-semi-continuous if $f^{-1}(V)$ is (γ,γ')-semi-open in X for every $V \in \sigma_{(\beta,\beta)'}$,

(4) $(\gamma,\gamma')\text{-}(\beta,\beta')$-precontinuous if $f^{-1}(V)$ is (γ,γ')-preopen in X for every $V \in \sigma_{(\beta,\beta)'}$,

(5) (γ,γ')-(β,β')-$\mathcal{A}$-continuous if $f^{-1}(V)$ is (γ,γ')-$\mathcal{A}$-open in X for every $V \in \sigma_{(\beta,\beta)}$,

(6) (γ,γ')-(β,β')-$\mathcal{B}$-continuous [4] if $f^{-1}(V)$ is (γ,γ')-$\mathcal{B}$-open in X for every $V \in \sigma_{(\beta,\beta)}$.

3. Properties of $\mathcal{LC}$-sets

Definition 3.1 : A subset A of a bioperation-topological space (X,τ,γ,γ') is said to be a locally (γ,γ')-closed set if $A = G \cap H$ where G is a (γ,γ')-open and H is a (γ,γ')-closed. The set of all (γ,γ')-$\mathcal{LC}$-sets denoted by $\mathcal{LC}$.

Remark 3.2 :

(1) Every (γ,γ')-open set as well as a (γ,γ')-closed set is locally (γ,γ')-closed.

(2) Finite intersection of locally (γ,γ')-closed sets is locally (γ,γ')-closed.

Theorem 3.3 : *Let (X,τ,γ,γ') be a bioperation-topological space and $N \subset X$. Then $N \in \mathcal{LC}$ if, and only if there exists a (γ,γ')-open set G such that $N = G \cap (\gamma,\gamma')\text{-}Cl(N)$.*

Proof: Let N be a (γ,γ')-closed set of (X,τ,γ,γ'). Then $N = G \cap A$ where G is (γ,γ')-open and A is (γ,γ')-closed in (X,τ). We have $(\gamma,\gamma')\text{-}Cl(S) \subset A$. Thus, $N = N \cap (\gamma,\gamma')\text{-}Cl(N) = G \cap A \cap (\gamma,\gamma')\text{-}Cl(N) = G \cap (\gamma,\gamma')\text{-}Cl(N)$. Let $N = G \cap (\gamma,\gamma')\text{-}Cl(N)$ where G is a (γ,γ')-open in (X,τ,γ,γ'). Then N is (γ,γ')-closed since $(\gamma,\gamma')\text{-}Cl(N)$ is (γ,γ')-closed. □

Theorem 3.4 : *Let (X,τ,γ,γ') be a bioperation-topological space and $K \subset L \subset X$. If L is locally (γ,γ')-closed, then there exists a locally (γ,γ')-closed set M such that $K \subset M \subset L$.*

Proof: Since L is (γ,γ')-closed, $L = S \cap (\gamma,\gamma')\text{-}Cl(L)$ where S is (γ,γ')-open. Since $K \subset L$, then $K \subset S$ and $K \subset (\gamma,\gamma')\text{-}Cl(K)$. We have $K \subset S \cap (\gamma,\gamma')\text{-}Cl(K)$. If we take $M = S \cap (\gamma,\gamma')\text{-}Cl(K)$, hence $K \subset M \subset L$. □

Theorem 3.5 : *Let (X,τ,γ,γ') be a bioperation-topological space and $K \subset X$. If K is a locally (γ,γ')-closed set, then there exists a (γ,γ')-closed set L in X such that $K \cap L = \varnothing$.*

Proof: Let $K = M \cap N$ where M is (γ,γ')-open and N is (γ,γ')-closed in (X,τ,γ,γ'). Suppose $L = N \setminus M$. Hence L is (γ,γ')-closed and $K \cap L = \varnothing$. □

Theorem 3.6 : *Let (X,τ,γ,γ') be a bioperation-topological space and let $A,B \subset X$ such that A is (γ,γ')-open and B is (γ,γ')-closed in X. Then there exists a (γ,γ')-open set C and a (γ,γ')-closed set D such that $A \cap B \subset D$ and $C \subset A \cup B$.*

Proof : Let $D = (\gamma,\gamma')\text{-}Cl(A) \cap B$ and $C = A \cup (\gamma,\gamma')\text{-}Int(B)$. Then D is (γ,γ')-closed and C is (γ,γ')-open. $A \subset (\gamma,\gamma')\text{-}Cl(A)$ implies $A \cap B \subset D$ and $(\gamma,\gamma')\text{-}Int(B) \subset B$ implies $C \subset A \cup B$. Hence, $A \cap B \subset D$ and $C \subset A \cup B$. □

Theorem 3.7 : *For a bioperation-topological space (X,τ,γ,γ'), the following properties are equivalent:*

(1) $A \in \mathcal{LC}$,

(2) $A = T \cap (\gamma,\gamma')\text{-}Cl(A)$ for some (γ,γ')-open set T,

(3) $(\gamma,\gamma')\text{-}Cl(A) \setminus A$ is (γ,γ')-closed,

(4) $A \cup (X \setminus (\gamma,\gamma')\text{-}Cl(A))$ is (γ,γ')-open,

(5) $A \subset (\gamma,\gamma')\text{-}Int(A \cup (X \setminus (\gamma,\gamma')\text{-}Cl(A)))$.

Proof : (1) $\Leftrightarrow$ (2): It follows from Theorem 3.3.

(2) $\Rightarrow$ (3): $(\gamma,\gamma')\text{-}(\gamma,\gamma')\text{-}Cl(A) \cap (X \setminus A) = (\gamma,\gamma')\text{-}Cl(A) \cap (X \setminus T)$ which is (γ,γ')-closed.

(3) $\Rightarrow$ (4): $A \cup (X \setminus (\gamma,\gamma')\text{-}Cl(A)) = X \setminus ((\gamma,\gamma')\text{-}Cl(A) \setminus A)$. Hence $A \cup (X \setminus (\gamma,\gamma')\text{-}Cl(A))$ is a (γ,γ')-open.

(4) $\Rightarrow$ (5): Since $A \cup (X \setminus (\gamma,\gamma')\text{-}Cl(A))$ is (γ,γ')-open, $A \subset (\gamma,\gamma')\text{-}Int(A \subset (X \setminus (\gamma,\gamma')\text{-}Cl(A)))$.

(5) $\Rightarrow$ (1): Since $A \subset (\gamma,\gamma')\text{-}Int(A \cup (X \setminus (\gamma,\gamma')\text{-}Cl(A)))$, then $A = (\gamma,\gamma')\text{-}Int(A \cup (X \setminus (\gamma,\gamma')\text{-}Cl(A))) \cap ((\gamma,\gamma')\text{-}Cl(A))$. Hence, $A \in \mathcal{LC}$. □

Theorem 3.8 : *A subset F of a bioperation-topological space (X,τ,γ,γ') is a $(\gamma,\gamma')\text{-}\mathcal{A}$-set if, and only if it is both (γ,γ')-semi-open and $(\gamma,\gamma')\text{-}\mathcal{LC}$-set.*

Proof : Let F be a $(\gamma,\gamma')\text{-}\mathcal{A}$-set, so $F = G \cap H$ where $G \in \tau_{(\gamma,\gamma')}$ and $H = (\gamma,\gamma')\text{-}Cl((\gamma,\gamma')\text{-}Int(H))$. Then $F \in \mathcal{LC}$. Now $(\gamma,\gamma')\text{-}Int(F) = (\gamma,\gamma')\text{-}Int(G) \cap (\gamma,\gamma')\text{-}Int(H) = G \cap (\gamma,\gamma')\text{-}Int(H)$, so that $F = G \cap (\gamma,\gamma')\text{-}Cl((\gamma,\gamma')\text{-}Int(H)) \subset (\gamma,\gamma')\text{-}Cl(G \cap (\gamma,\gamma')\text{-}Int(H)) = (\gamma,\gamma')\text{-}Cl((\gamma,\gamma')$

$-Int(F))$. Therefore, F is a (γ,γ')-semi-open set. Conversely, let F be (γ,γ')-semi-open and $\mathcal{LC}$-set, so that $F \subset (\gamma,\gamma')-Cl((\gamma,\gamma')-Int(F))$ and $F = G \cap (\gamma,\gamma')-Cl(F)$ where $G \in \tau_{(\gamma,\gamma')}$. Then $(\gamma,\gamma')-Cl(F) = (\gamma,\gamma')-Cl((\gamma,\gamma')-Int(F))$ and $(\gamma,\gamma')-Cl(F) = (\gamma,\gamma')-Cl((\gamma,\gamma')-Cl(F)) = (\gamma,\gamma')-Cl((\gamma,\gamma')-Int((\gamma,\gamma')-Cl(F)))$, so $(\gamma,\gamma')-Cl(F)$ is (γ,γ')-regular closed. Hence F is a $(\gamma,\gamma')-\mathcal{A}$-set. □

The following examples show that (γ,γ')-semi-open sets and $(\gamma,\gamma')-\mathcal{LC}$-sets are independent of each other.

Example 3.9 : Let $X = \{a,b,c\}$ and $\tau = \{\emptyset,\{a\},\{b\},\{a,b\},X\}$. We define the operations $\gamma,\gamma' : \tau \to \mathcal{P}(X)$ as follows

$$A^{\gamma} = \begin{cases} A & \text{if } a \in A, \\ A \cup \{a\} & \text{if } a \notin A, \end{cases} \text{ and } A^{\gamma'} = \begin{cases} A & \text{if } a \in A, \\ Cl(A) & \text{if } a \notin A. \end{cases}$$

Then $\{b\}$ is a $(\gamma,\gamma')-\mathcal{LC}$-set but not (γ,γ')-semi-open. Also $\{a,c\}$ is (γ,γ')-semi-open but not $(\gamma,\gamma')-\mathcal{LC}$-set.

Theorem 3.10 : *For a bioperation-topological space (X,τ,γ,γ'), the following properties are equivalent:*

(1) *F is a (γ,γ')-open.*
(2) *F is both $(\gamma,\gamma')-\alpha$-open and $(\gamma,\gamma')-\mathcal{LC}$-set.*
(3) *F is both (γ,γ')-pre-open and $(\gamma,\gamma')-\mathcal{LC}$-set.*

Proof : (1) $\Rightarrow$ (2) $\Rightarrow$ (3): Obvious.

(3) $\Rightarrow$ (1): Let F be (γ,γ')-pre-open and $(\gamma,\gamma')-\mathcal{LC}$-set, so that $F \subset (\gamma,\gamma')-Int((\gamma,\gamma')-Cl(F))$ and $F = G \cap (\gamma,\gamma')-Cl(F)$. Then $F \subset G \cap (\gamma,\gamma')-Int((\gamma,\gamma')-Cl(F)) = (\gamma,\gamma')-Int(G \cap (\gamma,\gamma')-Cl(F)) = (\gamma,\gamma')-Int(F)$, hence F is (γ,γ')-open.

Remark 3.11 : For a bioperation-topological space (X,τ,γ,γ'), we have the following

(1) $\mathcal{A} = s\tau_{(\gamma,\gamma')} \cap \mathcal{LC}$.
(2) $\tau_{(\gamma,\gamma')} = \alpha\tau_{(\gamma,\gamma')} \cap \mathcal{LC}$.
(3) $\tau_{(\gamma,\gamma')} = p\tau_{(\gamma,\gamma')} \cap \mathcal{LC}$.
(4) $\tau_{(\gamma,\gamma')} = p\tau_{(\gamma,\gamma')} \cap \mathcal{A}$.
(5) $\alpha\tau_{(\gamma,\gamma')} = p\tau_{(\gamma,\gamma')} \cap s\tau_{(\gamma,\gamma')}$.

Definition 3.12 : A subset A of a bioperation-topological space (X,τ,γ,γ') is said to be generalized (γ,γ')-closed if $(\gamma,\gamma')\text{-}Cl(A)\subset U$ whenever $A\subset U$ and U is (γ,γ')-open in X. The complement of a generalized (γ,γ')-closed set is called a generalized (γ,γ')-open set.

Theorem 3.13 : *A subset A is (γ,γ')-closed if and only if it is generalized (γ,γ')-closed and locally (γ,γ')-closed.*

Proof : Suppose that A is a (γ,γ')-closed set in X. Let $A\subset U$ where U is (γ,γ')-open in X. Then $(\gamma,\gamma')Cl(A)=A\subset U$. Thus A is generalized (γ,γ')-closed. Since A is (γ,γ')-closed it is locally (γ,γ')-closed. Conversely suppose that A is generalized (γ,γ')-closed and locally (γ,γ')-closed. Thus $A=U\cap F$, where U is (γ,γ')-open and F is (γ,γ')-closed. So $A\subset U$ and $A\subset F$. So by hypothesis $(\gamma,\gamma')\text{-}Cl(A)\subset U$ and $(\gamma,\gamma')\text{-}Cl(A)\subset(\gamma,\gamma')\text{-}Cl(F)=F$. Thus $(\gamma,\gamma')\text{-}Cl(A)\subset U\cap F=A$. Thus A is (γ,γ')-closed. □

Proposition 3.14 : Every locally (γ,γ')-closed set is a $(\gamma,\gamma')\text{-}\mathcal{B}$-set.

Proof : Let A be a locally (γ,γ')-closed subset of X. Then $A=U\cap F$, where U is (γ,γ')-open in X and F is (γ,γ')-closed. Then $A=(\gamma,\gamma')\text{-}Cl(A)$. Thus $(\gamma,\gamma')\text{-}Int(A)=(\gamma,\gamma')\text{-}Int((\gamma,\gamma')\text{-}Cl(A))$. Therefore A is a (γ,γ')-t-set. Hence A is a $(\gamma,\gamma')\text{-}\mathcal{B}$-set. □

The following example, shows that the converse of Proposition 3.14 is not true.

Example 3.15 : Let $X=\{a,b,c\}$ and $\tau=\{\varnothing,\{a\},\{b\},\{a,b\},\{a,c\},X\}$. We define the operations $\gamma,\gamma':\tau\to\mathcal{P}(X)$ as follows

$$A^{\gamma}=\begin{cases}A & \text{if } b\notin A,\\ Cl(A) & \text{if } b\in A,\end{cases}\quad\text{and}\quad A^{\gamma'}=\begin{cases}Cl(A) & \text{if } b\notin A,\\ A & \text{if } b\in A.\end{cases}$$

Clearly, $\{b,c\}$ is $(\gamma,\gamma')\text{-}\mathcal{B}$-set but not (γ,γ')-closed set.

4. Properties of $\mathcal{LC}$-continuity

Definition 4.1 : A function $f:(X,\tau,\gamma,\gamma')\to(Y,\sigma,\beta,\beta')$ is said to be $(\gamma,\gamma')\text{-}(\beta,\beta')\text{-}\mathcal{LC}$-continuous if $f^{-1}(V)$ is $(\gamma,\gamma')\text{-}\mathcal{LC}$-set in X for every $V\in\sigma_{(\beta,\beta)}$.

Theorem 4.2 : *Every $(\gamma,\gamma')\text{-}(\beta,\beta')\text{-}\mathcal{A}$-continuous function is $(\gamma,\gamma')\text{-}(\beta,\beta')\text{-}\mathcal{LC}$-continuous.*

Proof : Obvious. □

The following example shows that the converse of Theorem 4.2 is not true.

Example 4.3 : Let $X = \{a,b,c\}$, $\tau = \{\varnothing,\{a\},\{b\},\{a,b\},X\}$ and $\sigma = \{\varnothing,\{b\},X\}$. We define the operations $\gamma,\gamma' : \tau \to \mathcal{P}(X)$ as follows

$$A^{\gamma} = \begin{cases} A & \text{if } a \in A, \\ A \cup \{a\} & \text{if } a \notin A, \end{cases} \text{ and } A^{\gamma'} = \begin{cases} A & \text{if } a \in A, \\ Cl(A) & \text{if } a \notin A. \end{cases}$$

We define the operations $\beta,\beta' : \tau \to \mathcal{P}(X)$ as follows

$$A^{\beta} = \begin{cases} A & \text{if } b \in A, \\ Cl(A) & \text{if } b \notin A, \end{cases} \text{ and } A^{\beta'} = \begin{cases} A & \text{if } b \in A, \\ A \cup \{b\} & \text{if } b \notin A. \end{cases}$$

Then the identity function $f : (X,\tau,\gamma,\gamma') \to (Y,\sigma,\beta,\beta')$ is a (γ,γ')-(β,β')-$\mathcal{LC}$-continuous but not (γ,γ')-(β,β')-$\mathcal{A}$-continuous.

Proposition 4.4 : Every (γ,γ')-(β,β')-$\mathcal{LC}$-continuous function is (γ,γ')-(β,β')-$\mathcal{B}$-continuous.

Proof: The proof follows from Proposition 3.14. □

Theorem *For a function $f : (X,\tau,\gamma,\gamma') \to (Y,\sigma,\beta,\beta')$, we have the following*

(1) *f is a (γ,γ')-(β,β')-$\mathcal{A}$-continuous if, and only if it is both (γ,γ')-(β,β')-semi-continuous and (γ,γ')-(β,β')-$\mathcal{LC}$-continuous.*

(2) *f is a (γ,γ')-(β,β')-continuous if, and only if it is both (γ,γ')-(β,β')-α-continuous and (γ,γ')-(β,β')-$\mathcal{LC}$-continuous.*

(3) *f is a (γ,γ')-(β,β')-continuous if, and only if it is both (γ,γ')-(β,β')-precontinuous and (γ,γ')-(β,β')-$\mathcal{LC}$-continuous.*

(4) *f is a (γ,γ')-(β,β')-continuous if, and only if it is both (γ,γ')-(β,β')-precontinuous and (γ,γ')-(β,β')-$\mathcal{A}$-continuous.*

(5) *f is a (γ,γ')-(β,β')-α-continuous if, and only if it is both (γ,γ')-(β,β')-precontinuous and (γ,γ')-(β,β')-semi-continuous.*

Proof : The proof follows from Remark 3.11. □

Theorem 4.6 : *A function $f : (X,\tau,\gamma,\gamma') \to (Y,\sigma,\beta,\beta')$ is a (γ,γ')-(β,β')-$\mathcal{A}$-continuous if, and only if it is both (γ,γ')-(β,β')-semi-continuous and (γ,γ')-(β,β')-$\mathcal{LC}$-continuous.*

Proof : The proof follows from Theorem 3.8. □

Theorem 4.7 *For a function* $f:(X,\tau,\gamma,\gamma')\rightarrow(Y,\sigma,\beta,\beta')$, , *the following properties are equivalent:*

(1) *f is a* (γ,γ')-(β,β')*-continuous.*

(2) *f is both* (γ,γ')-(β,β')-α*-continuous and* (γ,γ')-(β,β')-$\mathcal{LC}$ *-continuous.*

(3) *f is both* (γ,γ')-(β,β')*-precontinuous and* (γ,γ')-(β,β')-$\mathcal{LC}$ *-continuous.*

Proof: The proof follows from Theorem 3.10. □

Conclusion

(1) The locally (γ,γ')-closed sets are defined, studied and characterized.

(2) The (γ,γ')-open sets are characterized.

(3) The (γ,γ')-(β,β')-$\mathcal{LC}$ -continuous functions are defined, studied, characterized and some relationships with another well known (γ,γ')-(β,β')-continuous functions are studied..

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