



## Optimizing the Multi-Level Location-Assignment Problem in Queue Networks Using a Multi-Objective Optimization Approach

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**Abstract.** Using hubs in distribution networks is an efficient approach. In this paper, a model for the location-allocation problem is designed within the framework of the queuing network in which services have several levels, and customers must go through these levels to complete

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the service. The purpose of the model is to locate an appropriate number of facilities among potential locations and allocate customers. The model is presented as a multi-objective nonlinear mixed-integer programming model. The objective functions include the summation of the customer and the waiting time in the system and the waiting time in the system and minimizing the maximum possibility of unemployment in the facility. To solve the model, the technique of accurate solution of the epsilon constraint method is used for multi-objective optimization, and Pareto solutions of the problem will be calculated. Moreover, the sensitivity analysis of the problem is performed, and the results demonstrate sensitivity to customer demand rate. Based on the results obtained, it can be concluded that the proposed model is able to greatly summate the customer and the waiting time in the system and reduce the maximum probability of unemployment at several levels of all facilities. The model can also be further developed by choosing vehicles for each customer.

**Keywords:** Location-Assignment, Hub, Reinforced Epsilon Constraint Method, Multilevel Services, Queue Theory, Multi-Objective Optimization

## **1. Introduction**

In 1909, Alfred Weber put forth the first developed general theory of industrial location, developed since the mid-1960s [19]. The term “location analysis” refers to the modeling, formulating, and solving a set of problems described as placing facilities in an assumed space. In this regard, deployment, positioning, location, and site selection are synonyms used for such issues [24]. Facility location includes determining a place for certain facilities to meet the demand of a set of customers. A branch of optimization knowledge, facility location problem is among the long-term and strategic decisions of various industrial systems and service centers [34-37]. However, it has less flexibility and must be considered in the early design stages due to its high costs [18]. Facility location plays a vital role in the performance of service systems. Particular attention should be paid to proper site selection for construction facilities. Focusing on customers and their satisfaction has always been one of the main activities of a service system [2]. The location-allocation problem is an essential set of facility location problems developed by Cooper in 1963 [8].

Fakhrzad presented a hub median location problem with multiple allocations of non-hub nodes to hub nodes [28]. Overall, it is possible to directly connect non-hub nodes with a penalty coefficient. These models locate facilities, allocate customer demands simultaneously, and sometimes determine facilities' optimal number and capacity [13,14,30,31]. Fakhrzad research [28] contributes to the design of green reverse SCN and the mentioned decisions as a location-routing-inventory model. His proposed model minimizes the total costs of chain and lost demands simultaneously and uses a fuzzy multi-objective solution approach. The concepts of reverse supply chain with location-routing-inventory model and simultaneous consideration of pickup and delivery activities have been used for perishable products among the first studies [32-34]. The proposed model's efficiency has been assessed using a bread production and distribution chain in Alborz province. Finally, the results show that the proposed model is applicable and efficient for the presented case study [29]. Facilities are hierarchical in most systems [10]. Moreover, a multi-level location problem or hierarchal hub location problem is proposed when various facilities provide different services. Communication and connection between different levels make it impossible to solve location problems independently for each level [28-30].

The multi-level systems are standard in various public and private sectors [16]. The queuing theory is one of the oldest analytical methods for modeling waiting queues [22]. Johnson published the first article on this topic in 1907, but his technique was mathematically inaccurate. Therefore, in terms of functional accuracy, Erlang's paper presented in 1909 was historically an essential part of the emergence of this area [7]. In many real-world location problems, there is a considerable time gap between service time and frequent visits of customers, and queues and congestion are inevitable [1]. In 1974, Larson considered the congestion phenomenon in location problems for the first time [15]. Congestion occurs when a service center cannot meet all simultaneous demands of customers [16]. According to Cooper, the queuing theory is an appropriate method for studying congested facilities [9]. In research, Wang et al. developed a single-objective model for location-allocation of bank automated teller machines (ATMs) within the framework of an M/M/1 queuing system. A limit is considered for the maximum allowable waiting time. Moreover, three heuristic algorithms are developed: a greedy-dropping procedure, a tabu search approach, and an  $\epsilon$ -optimal branch-and-bound method [27].

Pasandideh and Niaki focused on multi-objective modeling of the location-allocation problem while considering the M/M/M queue system. Using a genetics algorithm, they solved the mentioned model with two objectives of minimizing the average customer waiting time and minimizing the unemployment percentage in the facility [20]. In addition, they evaluated a multi-objective location-allocation problem within the framework of M\*/M/1 queuing systems with group entrance. They applied the standard  $L_p$ -metric method to turn the model into a single-objective one and solve the problem by a genetics algorithm and simulated annealing [21]. In another research, Hajipour et al. proposed a multi-objective multi-layer facility location-allocation (MLFLA) model with congested facilities using M/M/1 queuing systems. They solved the model by applying multi-objective vibration-damping optimization (MOVDO) and the multi-objective harmony search algorithm (MOHSA). In addition, the proposed techniques were compared to the non-dominated sorting genetic algorithm (NSGA-II) and multi-objective simulated annealing (MOSA) [11].

Harewood aimed to model a location-allocation problem with partial coverage for locating emergency service ambulances. The main objectives included maximizing the population receiving coverage within a given distance standard and with a given level of reliability and minimizing the cost of covering the population. Each facility was considered in the form of an M/M/m/k queuing model [29-31]. The minimum number of servers needed to cover point demand was calculated by specifying the minimum level of reliability and calculating the employee engagement probability [12,36,37]. Tavakkoli-Moghaddam et al. presented a multi-objective model for a location-allocation-pricing problem within the framework of the M/M/m/k queuing system, in which the number of servers in each facility, as well as its pricing and capacity, are parts of the variables of the model. In addition, different customer price sensitivity levels and facility distances were considered in the previous research. In the end, they developed a new multi-objective optimization algorithm based on a vibration theory, namely multi-objective vibration-damping optimization (MOVDO) to solve the problem and compared it with a non-dominated ranking genetic algorithm (NRGA) [26].

Syam proposed a single-objective nonlinear integer model for the location of available service systems in the form of the M/M/m queuing system. Various levels of order priority, various transportation modes, different servers, various working shifts, and different capacities are considered for numerous centers. The mentioned scholar linearized the model

and solved the equivalent model using the Lagrangian relaxation method [25]. Aboolian et al. developed a single-objective model for a network location-allocation problem in the form of the M/M/m queuing system. The number of servers in each facility was considered as decision variables of problem decision-making. Server allocation was primarily carried out through the greedy-dropping procedure, and two meta-heuristic algorithms, namely descent algorithm and genetics algorithm, were applied to solve the model [4]. In another study, Rahmati et al. developed a multi-objective location model within the M/M/m queuing system's framework with the variable of several servers. In this study, the researchers used MOHSA, NSGA-II, and NPGA [23]. Furthermore, Araz et al. established a single-objective location-allocation model for dispensing centers specific to an anthrax attack using M/G/m queues. In addition, a genetics algorithm was applied to solve the model [6].

The novelty of the study lies in the use of queuing networks and multi-level services. The present model proposes a new multi-objective model to solve a multi-level location-assignment problem of congestion-prone facilities. Different levels provide various services in this system, and service applicants must complete all levels to receive the benefit. In the current research, there is no going back to the point of demand. Considering congestion in this mode leads to shifting the problem's structure into a queue network because facilities of different levels depend on each other in terms of customer entry. The input of each facility at any level is the output of the lower-level facilities. Ultimately, the problem will be solved by a Pareto-based exact solution method. The schematic representation of the problem and mathematical modeling and description is provided in the second section of the article. The third section describes the solution method of the problem, and the fourth section presents computational results. Finally, the fifth section concludes by making suggestions for future research.

## **2. Statement of the Problem and Modeling**

There are different levels of services in the problem, each having a certain number of potential locations. The main objective is to select an appropriate number of facilities required at each level while taking limited budget and facility location simultaneously with customer allocation into account. Each customer goes through all levels to complete the order, receiving the desired service from one of the facilities at each level. Each facility behaves as an M/M/1 queuing system and forms a queuing network assuming the similarity of same-level facilities. Each service station can be considered as an independent M/M/1 model based on the theorem existing in queuing networks provided that there is infinite queue capacity in all stations and their efficiency coefficient is less than one. Accordingly, the sign of an entry is the same as the sign of customer entry [3]. Figure 1 depicts the queuing network of the multi-level location-allocation problem.

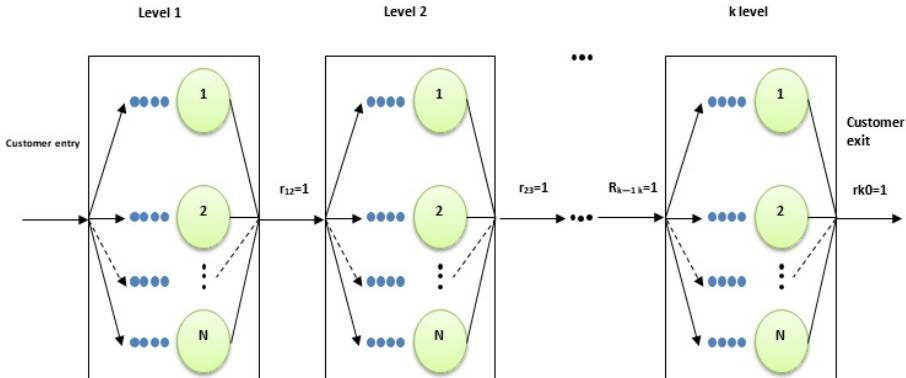


Figure 1. Queuing network of the multi-level location-allocation problem series

### 2.1. Model Premises

- The time gap between customer entry as well as service intervals is uncertain and independent of each other.
- The efficiency factor is less than one (condition for long-term system stability).
- The server is considered fixed and customers refer to these facilities to receive services.
- All facilities at the same level are similar and provide the same service.
- Various customers use similar vehicles.
- All customers must go through all levels and cannot leave the system in the middle levels (a necessary condition of referral).
- At each level, each customer is allocated to only one facility.
- There is a limited budget for facility construction.
- Customer demand has Poisson distribution, where the distribution parameter is the random number of triangular distribution.
- Serving duration has exponential distribution, where the distribution parameter is the random number of triangular distribution.
- The travel time between customer points and the first-level facilities and between the facilities of each level with higher levels is the random number of triangular distribution.

### 2.2. Model Parameters and Indexes

$i$  = Total demand points (customers);  $i=1,2,\dots,M$

$j'$  and  $j$  = total potential facility points at each level;  $j'=1,2,\dots,N$  and  $j=1,2,\dots,N$

$I$  = Facility level index;  $I=1,2,\dots,K$

$C_1$ = Construction cost of each facility at the  $l$ -th level

$B$ = Total facility construction budget

$\lambda_i$ = Customer demand rate at the  $i$ -th point

$\gamma_{jl}$ = Rate of customer entry into the  $j$ -th facility at the  $l$ -th level

$\mu_l$ = Service rate of each facility at the  $l$ -th level

$t_{ij}$ = Travel time from the  $i$ -th demand point to the  $j$ -th facility of the first level

$t'_{j'l}$ = Travel time from the  $j'$ -th facility at the  $l-1$  level to the  $j$ -th facility at the  $l$ -th level;  
 $l=2, \dots, K$

$\pi_{0jl}$ = Unemployment possibility (system vacancy in the  $i$ -th facility at the  $l$ -th level)

$\psi$ = A big positive number

$Z_1$ = Total travel time and total waiting time in the system

$Z_2$ = Maximum possibility of system idle mode among all facilities

### 2.3. Model Decision Variables

$y_{il}$ = Binary variable; 1, if the  $j$ -th facility of the  $l$ -th level is constructed; otherwise, 0.

$x_{ijl}$ = Binary variable; 1, if the  $i$ -th customer is allocated to the  $j$ -th facility at the  $l$ -th level; otherwise, 0.

### 2.4. Objective Functions and Constraints of the Problem

In this study, the model's objective functions include: 1) summation of the customer and the waiting time in the system, and 2) minimizing the maximum possibility of unemployment at various levels of all facilities. Notably, the objectives presented below are in conflict with each other:

$$\begin{aligned}
 \text{Min } Z_1 &= \sum_i^M = 1 \sum_j^N = 1 \lambda_i t_{ij} X_{ij1} + \sum_1^K = 2 \sum_i^M = 1 \sum_{j'}^N = 1 \lambda_i t'_{j'j1} X_{ij'}(I-1)X_{ijl} \\
 &+ \sum_I^K = 1 \sum_j^N = 1 \left( \frac{1}{\mu_I - \gamma_{jI}} \right) \gamma_{jI} \tag{1}
 \end{aligned}$$

$$\begin{aligned}
 \text{Min } Z_2 &= \max_{j = 1, \dots, N} \{ \pi_{0jI} \cdot y_{jI} \} \\
 I &= 1, \dots, K \tag{2}
 \end{aligned}$$

The mathematical model obtained after performing possible linearization is as follows. This model is a multi-objective nonlinear mixed integer programming program.

$$\begin{aligned}
 \text{Min } Z_1 &= \sum_{i=1}^M \sum_{j=1}^N \lambda_i t_{ij} X_{ij1} + \sum_{I=2}^K \sum_{i=1}^M \sum_{j=1}^N \sum_{j'=1}^N \lambda_i t'_{j'j1} u_{ij'jl} \\
 &+ \sum_{I=1}^K \sum_{j=1}^N \left( \frac{1}{\mu_I - \gamma_{jI}} \right) \gamma_{jI} \tag{3}
 \end{aligned}$$

$$\text{Min } Z_2 = W \tag{4}$$

**s.t:**

$$\begin{aligned}
 u_{ij'jl} &\leq X_{ij'(I-1)}; & i = 1, 2, \dots, M, j = 1, 2, \dots, N, j' = 1, 2, \dots, N, I \\
 &= 2, \dots, K & \tag{5}
 \end{aligned}$$

$$\begin{aligned}
 u_{ij'jl} &\leq \psi X_{ijl}; & i = 1, 2, \dots, M, j = 1, 2, \dots, N, j' = 1, 2, \dots, N, I \\
 &= 2, \dots, K & \tag{6}
 \end{aligned}$$

$$\begin{aligned}
 u_{ij'jl} &\geq X_{ij'(I-1)} - (1 - X_{ijl})\psi; & i = 1, 2, \dots, M, j = 1, 2, \dots, N, j' = 1, 2, \dots, N, I \\
 &= 2, \dots, K & \tag{7}
 \end{aligned}$$

$$\begin{aligned}
 W &\geq \pi_{0jI} y_{jI}; & I = 1, 2, \dots, K, j \\
 &= 1, 2, \dots, N & \tag{8}
 \end{aligned}$$

$$\begin{aligned}
 \sum_{j=1}^N y_{jI} &\geq 1; & I = 1, 2, \dots, K \tag{9}
 \end{aligned}$$

$$\begin{aligned}
 y_{jI} &\leq \sum_{i=1}^M X_{ijl} \leq M y_{jI}; & I = 1, 2, \dots, K, j \\
 &= 1, 2, \dots, N & \tag{10}
 \end{aligned}$$

$$\sum_{j=1}^N X_{ijl} = 1; \quad I = 1, 2, \dots, K, i = 1, 2, \dots, M \quad (11)$$

$$\sum_{j'=1}^N \sum_{j=1}^N u_{ij'jl} = 1; \quad I = 2, \dots, K, i = 1, 2, \dots, M \quad (12)$$

$$\sum_{I=1}^K \sum_{j=1}^N C_I y_{jl} \leq B; \quad (13)$$

$$\sum_{i=1}^M \lambda_i X_{ijl} \leq \mu_I; \quad I = 1, 2, \dots, K, j = 1, 2, \dots, N \quad (14)$$

$$Y_{jl} = \sum_{i=1}^M \lambda_i X_{ijl}; \quad I = 1, 2, \dots, K, j = 1, 2, \dots, N \quad (15)$$

$$\pi_{0jl} = 1 - \frac{Y_{jl}}{\mu_I}; \quad I = 1, 2, \dots, K, j = 1, 2, \dots, N \quad (16)$$

$$X_{ijl} \in \{0,1\}; \quad I = 1, 2, \dots, M, j = 1, 2, \dots, N, I = 1, 2, \dots, K \quad (17)$$

$$y_{jl} \in \{0,1\}; \quad j = 1, 2, \dots, N, I = 1, 2, \dots, K$$

$$u_{ij'jl} \in \{0,1\}; \quad i = 1, 2, \dots, M, \\ j = 1, 2, \dots, N, j' = 1, 2, \dots, N, I = 2, \dots, K$$

$$W \geq 0;$$

The objective function (3) summates the customer and the waiting time in the system, whereas objective function (4) minimizes the maximum possibility of unemployment at various levels of all facilities. Equations (5-8) are related to the linearization of objective functions. In addition, constraints (9) guarantee that at least one facility is constructed at each service level. Constraint (10) expresses that the customer must be only allocated to the constructed facilities, and a minimum of one customer and maximum to the total number of customers must be allocated to each constructed facility. Constraints (11 and 12) guarantee that each customer receives only one facility at each service level, and all customers must go through all levels. Constraints (13) show a budget limit for facility construction, whereas constraints (14) guarantee that the entry rate into each facility does not exceed the service capacity of that facility. Equation (15) expresses the entry rate into each facility, while Equation (16) calculates the unemployment probability in each facility at different levels. Ultimately, Constraints (17) determine the range of model decision variables.



### 3. Augmented $\epsilon$ -constraint Method

The augmented  $\epsilon$ -constraint method is one of the well-known approaches used to solve multi-objective issues. In this technique, one of the objective functions is optimized by transferring other objective functions to constraints. Despite the numerous advantages of this scientific method, compared to other approaches such as the weighting method, the following issues must be considered when the technique is applied: 1) calculating the range of objective functions in the series of tasks, 2) guaranteeing the applicability of the solution obtained, and 3) increasing the problem solution time in case of having more than two objective function. In order to eliminate the weaknesses of the  $\epsilon$ -constraint method, the lexicographic optimization is applied by the augmented  $\epsilon$ -constraint technique to calculate the values of the final result table. Another improvement is turning constraints related to the secondary objectives into equality by using covariates [17]. The augmented  $\epsilon$ -constraint method for minimization problems is presented below, where  $\delta$  is a small number and the gap between  $10^{-3}$  and  $10^{-6}$  and  $r_i$  is recognized as the range of objective function [5).

$$\text{Min } (f_1(\underline{x}) - \delta \times (s_2/r_2 + s_3/r_3 + \dots + s_k/r_k)) \tag{18}$$

subject to:

$$f_2(\underline{x}) + s_2 = \epsilon_2$$

$$f_3(\underline{x}) + s_3 = \epsilon_3$$

...

$$f_k(\underline{x}) + s_k = \epsilon_k$$

$$\underline{x} \in s, s, \in R^+$$

The augmented  $\epsilon$ -constraint method has five steps, including:

1. The values of the final results table are calculated using lexicographic optimization.
2. The best and worst values of each secondary objective function in the final result table (respectively, the maximum and minimum value of the corresponding column in the final result table in the maximization model) are extracted, and the range of each secondary objective function is calculated.
3. One of the objective functions is selected as the main objective function of the problem, and the other objective functions are placed in the constraints as a model [18].
4. The range of the secondary objective function is divided into a predetermined number (each of the resulting values is used for the  $f_i$  objective as one of the  $\epsilon_i$ s based on the desired number of Pareto solutions).

5. Model [18] is solved with the main objective function of the problem per each value of  $\epsilon_2, \dots, \epsilon_k$ , and the solutions obtained per each value of  $\epsilon_2, \dots, \epsilon_k$  are one of the Pareto solutions to the problem.

#### 4. Computational Results

In this section, a numerical example is presented and solved, and analyzed by the augmented  $\epsilon$ -constraint method in GAMS 24.12 software. In this example,  $M=5$ ,  $N=3$ ,  $K=2$ , and  $B=1200$ , and other values are randomly created according to Table 1. Given the higher importance of customer satisfaction, compared to the objectives of owners, the first objective function of the model is selected as the main objective function of the problem for the Augmented  $\epsilon$ -constraint method. Table 2 presents a set of Pareto solutions to the problem. It is notable that the values of the variables that are not in the table are equal to zero.

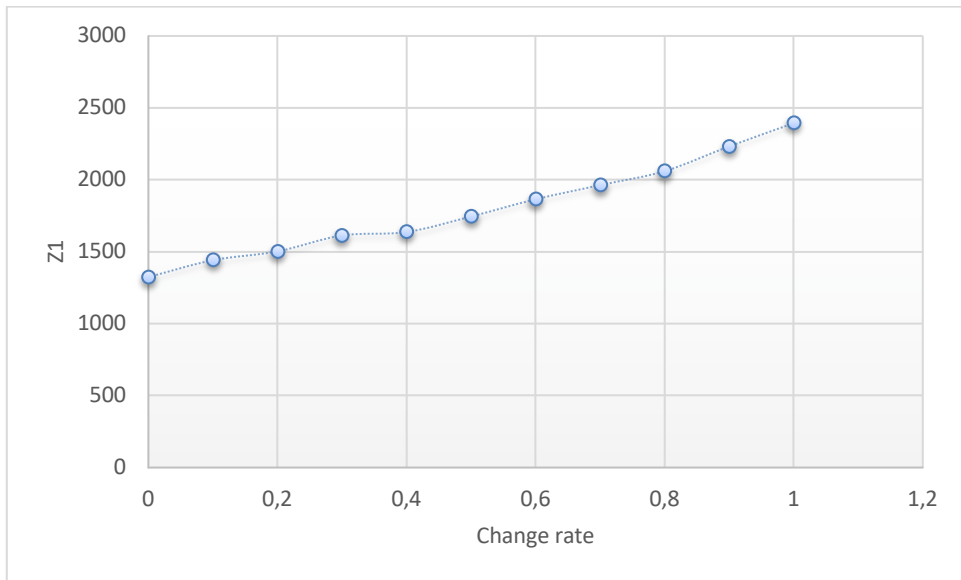
**Table 1. Distribution of random parameters of the random distribution problem**

Parameter	Random Distribution	Quantity Unit
$\lambda_i$	Triangular distribution with parameters (8/10/12)	People per time unit
$\mu_1$	Triangular distribution with parameters (30/40/50)	People per time unit
$\mu_2$	Triangular distribution with parameters (60/90/120)	People per time unit
$C_1$	Uniform distribution (100,500)	Monetary unit
$t_{1j}$	Triangular distribution with parameters (10,15/20)	Time unit
$t_{2j}$	Triangular distribution with parameters (15,17,5/20)	Time unit
$t_{3j}$	Triangular distribution with parameters (15/2, 0/25)	Time unit
$t_{4j}$	Triangular distribution with parameters (10/2, 0/30)	Time unit
$t_{5j}$	Triangular distribution with parameters (10, 14/18)	Time unit
$t'_{1j2}$	Triangular distribution with parameters (6/12/18)	Time unit
$t'_{2j2}$	Triangular distribution with parameters (7, 14/21)	Time unit
$t'_{3j2}$	Triangular distribution with parameters (9/18/27)	Time unit

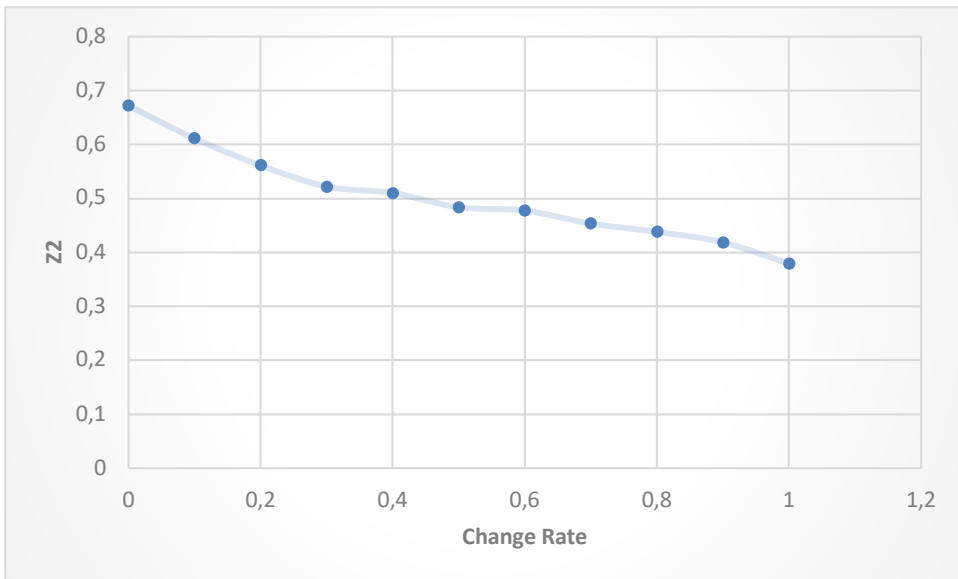
**Table 2. Model solution results by Augmented  $\epsilon$ -constraint method**

Solution Number	Objective Function Value	Location Variables value	Allocation Variables Value
Point 1	$f_1 = 1367.3$	$y_{11} = y_{21} = y_{12} = y_{22} = 1$	$x_{121} = x_{211} = x_{321} = x_{411}$ $= x_{511}$ $= x_{132}$ $= x_{212}$ $= x_{332}$ $= x_{412}$ $= x_{512} = 1$
	$f_2 = 0,786$		
Point 2	$f_1 = 1142.9$	$y_{11} = y_{21} = y_{12} = 1$	$x_{111} = x_{211} = x_{321} = x_{411}$ $= x_{511}$ $= x_{122}$ $= x_{212}$ $= x_{312}$ $= x_{412}$ $= x_{512} = 1$
	$f_2 = 0,728$		
Point 3	$f_1 = 163.75$	$y_{11} = y_{21} = y_{12} = 1$	$x_{121} = x_{211} = x_{321} = x_{411}$ $= x_{511}$ $= x_{112}$ $= x_{212}$ $= x_{312}$ $= x_{412}$ $= x_{512} = 1$
	$f_2 = 0,494$		

Sensitivity analysis is performed for the parameters to find the effective parameters in the model. In this problem, the demand rate parameter for several specific values for each objective function is solved and the results are shown in figures 2 and 3. As expected, as the demand rate increases, the total travel time of all customers and the waiting time in the system increases, and the maximum probability of unemployment decreases.

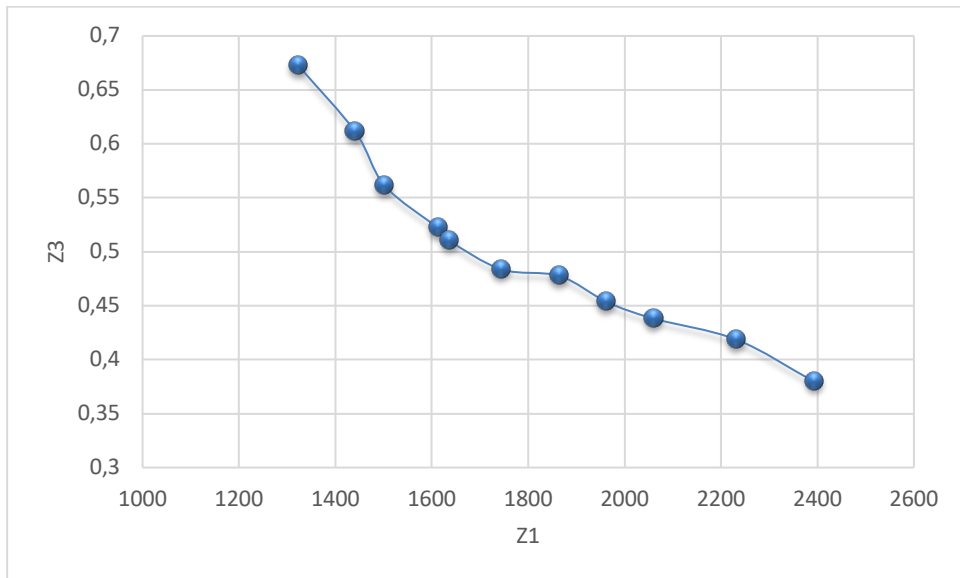


**Figure 2. Analysis of demand rate sensitivity to the first objective function**



**Figure 3. Analysis of demand rate sensitivity to the second objective function**

According to Figure 2, the uptrend in the first objective function indicates that as demand increases, the waiting time will increase as well. According to Figure 3, however, increasing demand reduces unemployment. The composition of this topic is presented in Figure 4, which shows a set of different modes between waiting times and idle times, which can be introduced as the Pareto border of the research problem.



**Figure 4. Pareto border of mathematical model solutions**

The analyses obtained from Figure 4 are indicative of a significant relationship between the first objective function and the second objective function. more servers should be included in the system as efforts are made to reduce idle time (first goal). This increases the idle time of the system (the second goal). This trend can be seen precisely in Figure 4.

## 5. Conclusion

The present article proposed a new model for a multi-level location-allocation problem, in which congestion in the system was considered within the framework of queuing networks. The problem was of queuing network type due to the multi-level structure of the issue and the same order of receiving services from different levels by the customer. The problem's goals were to summate the customer and the waiting time in the system and minimize the maximum possibility of unemployment at various levels of all facilities. To this end, a multi-objective nonlinear mixed integer programming model was presented and solved by a numerical example applying the augmented  $\epsilon$ -constraint method. Notably, the augmented  $\epsilon$ -constraint method is a Pareto-based exact solution method and has numerous advantages compared to other similar techniques. In addition, sensitivity analysis of the problem was carried out, and the results showed sensitivity to customer demand rate. In the present article, each facility was considered an M/M/1 queuing system. According to the study results, the main advantage of the proposed model is that the model is capable of summing the customer and the waiting time in the system and lowering the maximum probability of unemployment at several levels of all facilities. Hence, it is recommended that other queuing models be used in future studies. In addition, it is suggested that the server allocation problem be added to the model, meaning that the number of servers is considered a random variable

in each facility. Furthermore, the phenomenon of customer cancellation can also be considered in the problem.

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